Question 1: Let $\mathbf{A} = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$.

(a) Determine $\mathbf{A}^{-1}$ or show that it does not exist. Use any method you wish.

[7]

(b) Based on your result from part (a), how many solutions (x_1, x_2, x_3) does the system

$$\mathbf{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

have? Explain.

[3]

Question 2: Determine $\mathbf{A}$ if $(3\mathbf{I} - 2\mathbf{A}^{\mathsf{T}})^{-1} = \begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$.

[3]

Question 3: Let $\mathbf{B} = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$ and $\mathbf{F} = \begin{bmatrix} 8 & 1 & 5 \\ 8 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$. Find a matrix $\mathbf{E}$ so that $\mathbf{EF} = \mathbf{B}$.

[3]

Question 4: Show that

$$\mathbf{A} = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$$

is not invertible for any values of a, b, c, d, e, f, g, h .

[4]

Question 5: How many 2×2 diagonal matrices satisfy $\mathbf{A}^2 - 3\mathbf{A} - 4\mathbf{I} = \mathbf{0}$?

[3]

Question 6: Compute

$$\begin{vmatrix} 2 & 0 & 1 & 3 \\ 1 & 1 & 3 & 2 \\ 1 & 0 & -1 & 2 \\ 3 & -1 & 2 & 4 \end{vmatrix}$$

[7]

Question 7: Find the deteminant of the $n \times n$ matrix

$$\begin{bmatrix} (1-n) & 1 & 1 & \cdots & 1 \\ 1 & (1-n) & 1 & \cdots & 1 \\ 1 & 1 & (1-n) & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & (1-n) \end{bmatrix}$$

[4]

Question 8: Compute $\mathbf{A}^{-1}$ where $\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$. (Using adjoints here is likely easier.)

[3]

Question 9: If $\mathbf{A}$ and $\mathbf{B}$ are both 3×3 with $\det(\mathbf{A}) = 4$ and $\det(\mathbf{B}) = -3$, determine the value of $\det\left[(2\mathbf{A})^{-1}(5\mathbf{B})^{\mathsf{T}}\right]$.

[3]

Question 10: Use Cramer's rule to solve for z:

$$\begin{aligned} 4x + y + z &= 3 \\ 3x + 7y - z &= 0 \\ 7x + 3y - 5z &= 0 \\ x + z + 2w &= 1 \end{aligned}$$

[6]

Question 11: Let $\mathbf{A} = \begin{bmatrix} 1 & (k-1) & 7 \\ 2 & (k-3) & 4 \\ 5 & (k+1) & 0 \end{bmatrix}$. Find all value of k for which $\mathbf{A}$ is invertible.

[4]