Question 1: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} 2x - 6z &= -4 \\ 3x + y - 2z &= 5 \\ 2x + 2y + z &= 4 \end{aligned}$$

[10]

Question 2: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} 2x + 3z &= 3 \\ 4x - 3y + 7z &= 5 \\ 8x - 9y + 15z &= 10 \end{aligned}$$

[10]

Question 3: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} 3x + 3y + 12z &= 6 \\ x + y + 4z &= 2 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4 \end{aligned}$$

[10]

Question 4: Use matrices to determine the values of a, b and c so that the polynomial $y = ax^2 + bx + c$ passes through the points $(1, 4)$, $(2, 0)$ and $(3, 12)$.

[5]

Question 5: Determine all values of a (if any) for which the following system has exactly one solution:

$$x + 2y = 1$$
$$2x + (a^2 - 5)y = a - 1$$

[5]

Question 6: For this problem use the following matrices to carry out the indicated computations, if possible. If a given statement is not defined then state "not defined":

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

(a) $\mathbf{BC} - \mathbf{A}^{\mathsf{T}}$

[2]

(b) $\text{tr}(\mathbf{AC} - 4\mathbf{D}^{\mathsf{T}})$

[2]

(c) $7\mathbf{DE} + 5\mathbf{CA}$

[2]

(d) Determine the size of $\mathbf{ABCDE}$

[2]

(e) Find a matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ so that $\mathbf{B}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$.

[2]