In the following A, B and C are matrices while a, b and c are scalars. The sizes of matrices are such that each of the expressions is defined.

Basic Properties

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $A(BC) = (AB)C$
4. $A(B + C) = AB + AC$
5. $(B + C)A = BA + CA$
6. $A(B - C) = AB - AC$
7. $(B - C)A = BA - CA$
8. $a(B + C) = aB + aC$
9. $a(B - C) = aB - aC$
10. $(a + b)C = aC + bC$
11. $(a - b)C = aC - bC$
12. $a(bC) = (ab)C$
13. $a(BC) = (aB)C = B(aC)$

Zero Matrices

Definition: The **zero matrix** of size $m \times n$ is the matrix with each entry 0, denoted $\mathit{0}_{m\times n}$. The size is omitted when the context is clear.

Properties:

1. $A + \mathit{0} = \mathit{0} + A = A$
2. $A - \mathit{0} = A$
3. $A - A = \mathit{0}$
4. $\mathit{0}A = \mathit{0}$
5. If $cA = \mathit{0}$ then $c = 0$ or $A = \mathit{0}$

Identity Matrices

Definition: The **identity matrix** of order n is the **square** matrix with main diagonal entries of 1 and zeros elsewhere, denoted I_n. The size is omitted when the context is clear.

Example:

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Properties: Here assume A is $m \times n$:

1. $AI_n = A$
2. $I_m A = A$

Inverse of a Matrix

Definition: If A and B are $n \times n$ with the property that $AB = BA = I$, then B is called the **inverse** of A, written $B = A^{-1}$. That is,

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I$$

For a given square matrix A, if A^{-1} exists A is said to be **invertible** or **nonsingular**. If not invertible, A is said to be **singular**.