

**Question 1:** Let  $\mathbf{A} = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$ .

(a) Determine  $\mathbf{A}^{-1}$  or show that it does not exist. Use any method you wish.

[7]

(b) Based on your result from part (a), how many solutions  $(x_1, x_2, x_3)$  does the system

$$\mathbf{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

have? Explain.

[3]

**Question 2:** Determine  $\mathbf{A}$  if  $(3\mathbf{I} - 2\mathbf{A}^\mathsf{T})^{-1} = \begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$ .

[3]

**Question 3:** Let  $\mathbf{B} = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}$  and  $\mathbf{F} = \begin{bmatrix} 8 & 1 & 5 \\ 8 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$ . Find a matrix  $\mathbf{E}$  so that  $\mathbf{EF} = \mathbf{B}$ .

[3]

**Question 4:** Show that

$$\mathbf{A} = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$$

is not invertible for any values of  $a, b, c, d, e, f, g, h$ .

[4]

**Question 5:** How many  $2 \times 2$  diagonal matrices satisfy  $\mathbf{A}^2 - 3\mathbf{A} - 4\mathbf{I} = \mathbf{0}$  ?

[3]

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**Question 6:** Compute

$$\begin{vmatrix} 2 & 0 & 1 & 3 \\ 1 & 1 & 3 & 2 \\ 1 & 0 & -1 & 2 \\ 3 & -1 & 2 & 4 \end{vmatrix}$$

[7]

Question 7: Find the deteminant of the  $n \times n$  matrix

$$\begin{bmatrix} (1-n) & 1 & 1 & \cdots & 1 \\ 1 & (1-n) & 1 & \cdots & 1 \\ 1 & 1 & (1-n) & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & (1-n) \end{bmatrix}$$

[4]

Question 8: Compute  $\mathbf{A}^{-1}$  where  $\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$ . (Using adjoints here is likely easier.)

[3]

Question 9: If  $\mathbf{A}$  and  $\mathbf{B}$  are both  $3 \times 3$  with  $\det(\mathbf{A}) = 4$  and  $\det(\mathbf{B}) = -3$ , determine the value of  $\det[(2\mathbf{A})^{-1}(5\mathbf{B})^T]$ .

[3]

**Question 10:** Use Cramer’s rule to solve for  $z$ :

$$\begin{aligned} 4x + y + z &= 3 \\ 3x + 7y - z &= 0 \\ 7x + 3y - 5z &= 0 \\ x + z + 2w &= 1 \end{aligned}$$

[6]

**Question 11:** Let  $\mathbf{A} = \begin{bmatrix} 1 & (k - 1) & 7 \\ 2 & (k - 3) & 4 \\ 5 & (k + 1) & 0 \end{bmatrix}$ . Find all value of  $k$  for which  $\mathbf{A}$  is invertible.

[4]