Formula Sheet Policy

You may use a single letter-size double-sided "cheat-sheet" for the final exam. The cheat-sheet **must be handwritten** and may contain formulae, facts, definitions and theorems, however your cheat sheet **may not contain worked examples.** The instructor will have the final say on what is or is not appropriate for the cheat sheet. You must hand in your cheat sheet along with your completed exam. **To be considered for grading, your exam must include your cheat sheet.**

Calculator Policy

As with tests, you may use a basic scientific non-programmable, non-graphing calculator for the exam.

Final Exam Notes

Your final exam is comprehensive, covering all material since the first day of class. I recommend that you review all assigned homework and test practice material, and work through the term tests again (blank copies of the tests are posted on the course website.) In addition, you should work through the final exam from the Spring 2013 edition of the course; a copy is posted on our course website.

Must-know topics:

1. Gaussian Elimination: Know how to solve a system of m linear equations in n variables using Gaussian Elimination and/or Gauss-Jordan elimination. Know how to clearly reduce the augmented coefficient matrix to REF or RREF form and properly state the solution in the case that

   (a) there is exactly one solution

   (b) there are no solutions

   (c) there are infinitely many solutions (using parameters)

   Caution:

   (a) $m > n$ (more equations than variables) does not immediately indicate that there is no solution; reduce to REF or RREF to determine correct result.

   (b) Similarly, $m < n$ (fewer equations than variables) does not automatically imply infinitely many solutions; again, reduce to REF/RREF to determine result.

2. Matrix Algebra:

   (a) Know how to add, subtract, multiply matrices and find matrix inverses. Know how to identify when a matrix inverse does not exist.

   (b) Know what the trace and transpose of a matrix is.

(c) Know definitions of special matrices (identity, diagonal, upper/lower triangular, zero, symmetric.)

3. Determinants:

   (a) Know how to compute determinants by cofactor expansion.

   (b) Know the effect of row operations on determinants and how to use this to compute determinants of large matrices.

   (c) Know Cramer's Rule.

   (d) Know the basic algebraic properties of determinants (determinants of product, inverse, transpose.)

   (e) Know what the determinant tells us about the invertibility of square matrices and the number of solutions to the corresponding homogeneous system of equations.

4. Vectors in 2 and 3 space:

   (a) Know the basic vector algebra rules (Thm 3.1.1 and 3.1.2 in text.)

   (b) Know the definition of dot product and norm and how to use these to find the angle between two vectors. Know how to use the norm to find the distance between vectors.

   (c) Know what it means for vectors to be orthogonal and how to test for orthogonality using the dot product.

   (d) Know what a normal to a plane or line is and how to find equations of planes and lines using orthogonality.

   (e) Know the definition of vector projection and how to use it to find distance from points to planes, etc. Do not rely solely on the distance formulas in Thm 3.3.4: know why these work!

   (f) Know how to compute the cross product and the geometric relationship of $\mathbf{u} \times \mathbf{v}$ to $\mathbf{u}$ and $\mathbf{v}$.

   (g) Know the basic algebraic properties of cross products (Thm 3.5.2 of text.)

   (h) Know the geometric relationship between cross products and parallelograms (and parallelepipeds!)

5. General Vector Spaces

   (a) Know what a vector space is (no need to memorize or reproduce the axioms, however), and be comfortable with some examples of vector spaces: R^n, P_n, $F(-\infty, \infty)$, etc.

   (b) Know what a subspace is and how to test for it.

   (c) Know what the span of a set S is and that it defines a subspace. Know how to test a given set of vectors S to determine if it spans a given subspace. Caution: you cannot always use the determinant to test for spanning. For example, if S consists of 4 vectors from R^3, then you must go back to the definition to determine if these vectors span R^3.

   (d) Know the definition of linear independence and how to test for it. Caution: you cannot always use the determinant to test for linear independence; see Example 3 on page 192 of text.

   (e) Know how to test for linear independence of functions using the Wronskian.

   (f) Know the definition of basis and how to test whether a given set of vectors is a basis.

   (g) Know how to find the coordinates of a vector relative to a given basis.

   (h) Know the definition of dimension (the number of vectors in any basis for the vector space.)

(i) Know how to find a basis and the dimension of the solution space for a homogeneous system.

(j) Know how to find the transition matrix for a change of basis, and how the transition matrix can be used to change coordinates to reflect the new basis.

(k) Know the definition of row space, column space and null space of a matrix.

(l) Know how to find a basis and dimension for row, column and null spaces. Important examples: Example 4 on p.229, Example 6 on p.230, Example 7 on p.231.

(m) Know the definitions of rank and nullity and how to determine these quantities from the REF.

(n) Know what a matrix transformation is and how to find the standard matrix for a transformation.

(o) Know how to compose two transformations and how to use the standard matrix to find the image of a vector subject to projection, rotation, reflection, etc.

6. Applications:

(a) Know how to find the best fit line for given data (x_i, y_i) using the method of least squares—see notes and exercises posted on the course website.

(b) Know how to find the eigenvalue/eigenvector pairs associated with a given square matrix—see class notes and exercises posted on the course website.