Question 1: Evaluate the following integrals:

(a) $\int \sin^3(x)\cos^2(x)\,dx$

$= \int \sin^2(x)\cos^2(x)\sin(x)\,dx$

$= \int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx$ $\quad u = \cos(x)$, $du = -\sin(x)\,dx$

$= -\int (1-u^2)u^2\,du$

$= -\int u^2 - u^4\,du$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C$

$= \boxed{-\frac{\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C.}$

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(b) $\int \tan^4(x)\sec^4(x)\,dx$

$= \int \tan^4(x)\sec^2(x)\sec^2(x)\,dx$

$= \int \tan^4(x)(1+\tan^2(x))\sec^2(x)\,dx$ $\quad u = \tan(x)$, $du = \sec^2(x)\,dx$

$= \int u^4(1+u^2)\,du$

$= \int u^4 + u^6\,du$

$= \frac{u^5}{5} + \frac{u^7}{7} + C$

$= \boxed{\frac{\tan^5(x)}{5} + \frac{\tan^7(x)}{7} + C.}$

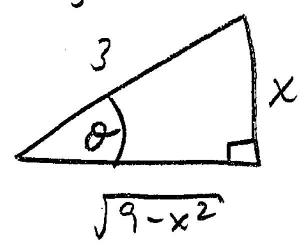
[5]

Question 2: Evaluate the integral

$$I = \int \frac{x^2}{\sqrt{9-x^2}}\,dx$$

Let $x = 3\sin\theta \longleftarrow \sin\theta = \frac{x}{3}$

$dx = 3\cos\theta\, d\theta$



$$I = \int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}}\, 3\cos\theta\, d\theta$$

$$= \int \frac{9\sin^2\theta}{\cancel{3}\sqrt{\cancel{1-\sin^2\theta}}}\, \cancel{3}\,\cancel{\cos\theta}\, d\theta$$

$$= 9\int \sin^2\theta\, d\theta$$

$$= 9\int \frac{1-\cos(2\theta)}{2}\, d\theta$$

$$= 9\int \frac{1}{2} - \frac{\cos(2\theta)}{2}\, d\theta$$

$$= 9\left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right] + C$$

$$= \frac{9}{2}\theta - \frac{9}{2}\sin(\theta)\cos(\theta) + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$$

$$= \boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C}$$

[10]

Question 3: Evaluate the integral

$$I = \int \frac{6}{x^3 + x^2 - 2x}\,dx$$

$$\frac{6}{x^3+x^2-2x} = \frac{6}{x(x^2+x-2)}$$

$$= \frac{6}{x(x-1)(x+2)}$$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$= \frac{A(x-1)(x+2) + Bx(x+2) + C(x)(x-1)}{x(x-1)(x+2)}$$

$$= \frac{(A+B+C)x^2 + (A+2B-C)x + (-2A)}{x(x-1)(x+2)}$$

$\therefore$ ① $A+B+C = 0$

② $A+2B-C = 0$

③ $-2A = 6$

③ $\Rightarrow A = -3$

$\therefore$ ① $\Rightarrow B + C = 3$

② $\Rightarrow 2B - C = 3$

$\therefore 3B = 6$

$\Rightarrow B = 2$

$\Rightarrow C = 1$

$$\therefore I = \int -\frac{3}{x} + \frac{2}{x-1} + \frac{1}{x+2}\,dx$$

$$= \boxed{-3\ln|x| + 2\ln|x-1| + \ln|x+2| + C.}$$

[10]

Question 4: Evaluate the following integrals:

(a) $\int x^3\sqrt{x^2+25}\,dx$ $= \frac{1}{2}\int x^2\sqrt{x^2+25}\;2x\;dx$ $\left\{\begin{array}{l} u = x^2+25 \\ du = 2x\;dx \end{array}\right.$

$$= \frac{1}{2}\int (u-25)\,u^{1/2}\,du$$

$$= \frac{1}{2}\int u^{3/2} - 25u^{1/2}\,du$$

$$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - 25\cdot\frac{2}{3}u^{3/2}\right] + C$$

$$= \boxed{\frac{1}{5}(x^2+25)^{5/2} - \frac{25}{3}(x^2+25)^{3/2} + C.}$$

[3]

(b) $I = \int \frac{x^3}{x^2-1}\,dx$

$u = x^2-1 \Rightarrow x^2 = u+1$

$du = 2x\;dx$

$$\therefore I = \frac{1}{2}\int \frac{u+1}{u}\,du$$

$$= \frac{1}{2}\int 1 + \frac{1}{u}\,du$$

$$= \frac{1}{2}\left[u + \ln|u|\right] + C = \boxed{\frac{1}{2}\left[x^2-1 + \ln|x^2-1|\right] + C.}$$

[3]

(c) $\int \cos^2(\pi t)\,dt$

$$= \int \frac{1+\cos(2\pi t)}{2}\,dt$$

$$= \boxed{\frac{t}{2} + \frac{\sin(2\pi t)}{4\pi} + C.}$$

[4]

Question 5: Use the trapezoid rule with $n = 4$ to approximate $\int_0^2 \left(x^4 - \frac{x^3}{2}\right)\,dx$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2 \quad \Big\} \; \Delta x = \frac{2-0}{4} = \frac{1}{2}$

x	$f(x) = x^4 - \frac{x^3}{2}$
0	0
$\frac{1}{2}$	0
1	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{54}{16} = \frac{27}{8}$
2	12

$$\therefore T_4 = \frac{(\frac{1}{2})}{2}\left[\cancel{(1)(0)+(2)(0)} + \cancel{(2)(\tfrac{1}{2})}^{\,1} + \cancel{(2)}\left(\frac{27}{\cancel{8}_{4}}\right) + (1)(12)\right]$$

$$= \left(\frac{1}{4}\right)\left[\frac{52+27}{4}\right]$$

$$= \boxed{\frac{79}{16}}$$

[5]

Question 6: The function $f(x)$ has the following values:

x	0	1/2	1	3/2	2
$f(x)$	-4	2	-2	a	-6

Notice in this table that $f(3/2) = a$. Using all of the data above and Simpson's Rule resulted in the approximation $\int_0^2 f(x)\,dx \approx 1$. Determine the value of a.

$$S_4 = \frac{\Delta x}{3}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$$

$$= \left[\frac{1}{6}\right]\left[(-4) + (4)(2) + (2)(-2) + (4)(a) + (-6)\right]$$

$$= \left[\frac{1}{6}\right]\left[4a - 6\right]$$

$$\therefore \left[\frac{1}{6}\right]\left[4a-6\right] = 1$$

$$4a - 6 = 6$$

$$a = \frac{6+6}{4} = \boxed{3}$$

[5]