

Question 1: Use the definition of the definite integral in the form

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

to evaluate

$$\int_1^2 (x^2 + 1) dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

Question 2: Water flows from a tank at a rate of $r(t) = 200 - 2t^2$ litres per minutes, where $0 \leq t \leq 10$. How much water flows from the tank over the first 10 minutes? State units with your answer.

[3]

Question 3: Find the number c in the interval $[2, 5]$ with the property that for $f(x) = (x - 3)^2$, $f_{\text{ave}} = f(c)$.

[3]

Question 4: Suppose f is a continuous function with the property

$$\int_0^x f(t) dt = xe^{2x} + \int_0^x e^{-t} f(t) dt .$$

Find a formula for $f(x)$.

[4]

Question 5: Evaluate the following integrals:

(a) $\int \left(\frac{\sqrt{x}}{2} - e^x + 1 \right) dx$

[3]

(b) $\int (x^{-1} - \csc(x) \cot(x)) dx$

[3]

(c) $\int_0^{\pi/4} (1 + \cos(x) + 2 \sec^2(x)) dx$

[4]

Question 6: Evaluate the following integrals:

(a) $\int x \sin(2x^2) dx$

[3]

(b) $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$

[3]

(c) $\int \frac{e^x}{e^x + e^{-x}} dx$

[4]

Question 7: Evaluate $\int_0^1 (1 - 3x)e^{2x} dx$

[5]

Question 8: Evaluate $\int x[\ln(x)]^2 dx$

[5]
