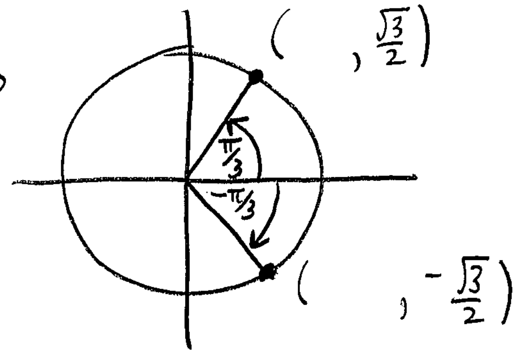
Question 1:

(a) Determine $\arcsin\left(-\sqrt{3}/2\right)$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Rightarrow$



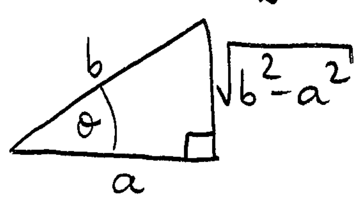
$\therefore \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$

[3]

(b) Determine $\csc\left(\cos^{-1}(a/b)\right)$. Your final answer should be a fraction which does not contain any trigonometric or inverse trigonometric functions.

Let $\theta = \cos^{-1}\left(\frac{a}{b}\right)$

$\therefore \cos(\theta) = \frac{a}{b}$:



$\therefore \csc\left(\cos^{-1}\left(\frac{a}{b}\right)\right)$

$= \csc(\theta)$

$= \boxed{\frac{b}{\sqrt{b^2-a^2}}}$

[3]

(c) Find all values of x at which the tangent line to $y = x\arctan(x/2) - \ln(x^2+4)$ has slope $\pi/4$.

Solve $y' = \pi/4$

$\Rightarrow \arctan\left(\frac{x}{2}\right) + x\frac{1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{1}{2} - \frac{1}{x^2+4}(2x) = \frac{\pi}{4}$

$\Rightarrow \arctan\left(\frac{x}{2}\right) + \frac{x}{2+\frac{x^2}{2}}\cdot\frac{2}{2} - \frac{2x}{x^2+4} = \frac{\pi}{4}$

$\Rightarrow \arctan\left(\frac{x}{2}\right) + \cancel{\frac{2x}{4+x^2}} - \cancel{\frac{2x}{x^2+4}} = \frac{\pi}{4}$

$\therefore \frac{x}{2} = 1 \Rightarrow \boxed{x = 2}$

[4]

Question 2:

(a) Let $f(x) = \cosh(\tanh(x))$. Calculate and simplify $f'(0)$.

$$f'(x) = \sinh(\tanh(x)) \cdot \operatorname{sech}^2(x)$$

$$f'(0) = \sinh(\cancel{\tanh(0)}) \cdot \frac{1}{\cancel{\cosh^2(0)}}$$

0, 1

$$= \boxed{0}$$

[2]

(b) Solve for x: $\sinh(x) = 2$.

$$\frac{e^{x} - e^{-x}}{2} = 2$$

$$e^x\left[e^x - e^{-x}\right] = [4]\, e^x$$

$$e^{2x} - 4e^x - 1 = 0$$

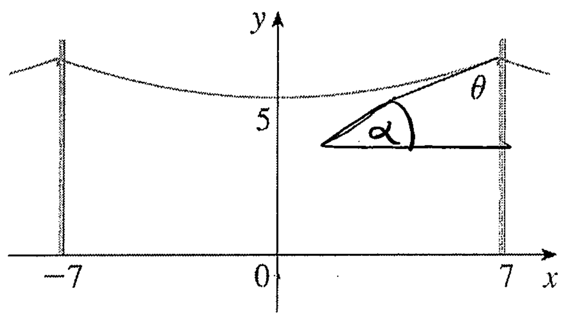
$$\therefore\ e^x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$\therefore\ \boxed{x = \ln(2+\sqrt{5})},\ \underbrace{\ln(2-\sqrt{5})}_{\text{not defined}}$$

[4]

(c) The cable hanging between the two poles in the figure below has equation $y = 20\cosh(x/20) - 15$. Determine the angle θ shown in the figure. Express your final answer as a "calculator ready" expression using hyperbolic and inverse trigonometric functions.



$$\tan(\alpha) = y'\Big|_{x=7} = 20\sinh\left(\frac{x}{20}\right)\left(\frac{1}{20}\right)\Big|_{x=7} = \sinh\left(\frac{7}{20}\right)$$

$$\therefore\ \alpha = \arctan\left(\sinh\left(\frac{7}{20}\right)\right)$$

$$\therefore\ \boxed{\theta = \frac{\pi}{2} - \arctan\left(\sinh\left(\frac{7}{20}\right)\right)}$$

[4]

Question 3: Find the following limits if they exist:

(a) $\lim_{x\to 0} \frac{8x^2}{\cos(x)-1}$ $\sim$ "$\frac{0}{0}$"

$\overset{H}{=} \lim_{x\to 0} \frac{16x}{-\sin(x)} \sim$ "$\frac{0}{0}$"

$\overset{H}{=} \lim_{x\to 0} \frac{16}{-\cos(x)} = \boxed{-16}$

[3]

(b) $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln(x)}$ $\sim$ "$\frac{-\infty}{-\infty}$"

$\overset{H}{=} \lim_{x\to 0^+} \frac{\left(\frac{2x+2}{x^2+2x}\right)}{\frac{1}{x}}$

$= \lim_{x\to 0^+} \frac{2(x+1)}{\cancel{x}(x+2)} \cdot \frac{\cancel{x}}{1}$

$= \frac{2}{2} = \boxed{1}$

[3]

(c) $\lim_{x\to 0^+} \left(\frac{3x+1}{x} - \frac{1}{\sin(x)}\right)$ $\sim$ "$\infty - \infty$"

$= \lim_{x\to 0^+} \frac{\sin(x)(3x+1)-x}{x\sin(x)} \sim$ "$\frac{0}{0}$"

$\overset{H}{=} \lim_{x\to 0^+} \frac{\cos(x)(3x+1)+\sin(x)(3)-1}{1\cdot\sin(x)+x\cos(x)} \sim$ "$\frac{0}{0}$"

$\overset{H}{=} \lim_{x\to 0^+} \frac{-\sin(x)(3x+1)+\cos(x)(3)+3\cos(x)}{\cos(x)+\cos(x)+x(-\sin(x))} = \frac{0+3+3}{1+1+0} = \boxed{3}$

[4]

Question 4: Find the following limit if it exists:

$$\lim_{x\to 0^+}(2x+x^2)^x \quad \sim \text{"}0^0\text{"}$$

$$(2x+x^2)^x = e^{x\ln(2x+x^2)}.$$

$$\lim_{x\to 0^+} x\ln(2x+x^2) \sim \text{"}0\cdot(-\infty)\text{"}$$

$$= \lim_{x\to 0^+} \frac{\ln(2x+x^2)}{\frac{1}{x}} \sim \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\left(\frac{2+2x}{2x+x^2}\right)}{\left(\frac{1}{x^2}\right)}$$

$$= \lim_{x\to 0^+} \frac{2(1+x)}{\cancel{x}(2+x)}\cdot\frac{x^{\cancel{2}}}{1} = 0$$

$$\therefore \lim_{x\to 0^+}(2x+x^2)^x = e^0 = \boxed{1}.$$

[5]

Question 5: Find $f(t)$ if $f''(t) = t - \cos(t)$ and $f'(0) = 2$, $f(0) = -2$.

$$f'(t) = \frac{t^2}{2} - \sin(t) + C_1$$

$$f'(0) = 2 \Rightarrow 2 = \frac{0^2}{2} - \sin(0) + C_1$$

$$\therefore C_1 = 2$$

$$\therefore f'(t) = \frac{t^2}{2} - \sin(t) + 2$$

$$f(t) = \frac{t^3}{6} + \cos(t) + 2t + C_2$$

$$f(0) = -2 \Rightarrow -2 = \cancel{\frac{0^3}{6}}^{0} + \cos(0) + \cancel{2(0)}^{0} + C_2$$

$$\therefore C_2 = -2 - 1 = -3.$$

$$\therefore f(t) = t^3/6 + \cos(t) + 2t - 3$$

[5]

Question 6: Find the most general antiderivative of each of the following functions:

(a) $f(x) = 5e^x + \dfrac{\cosh(x)}{2} + \sec^2(x)$

$$\boxed{F(x) = 5e^x + \frac{\sinh(x)}{2} + \tan(x) + C}$$

[3]

(b) $f(x) = \dfrac{1+\sqrt{x}+x^2}{x} = \dfrac{1}{x} + x^{-\frac{1}{2}} + x$

$$\boxed{F(x) = \ln|x| + 2x^{\frac{1}{2}} + \frac{x^2}{2} + C}$$

[3]

(c) $f(x) = \dfrac{2+x^2}{1+x^2}$ (Hint: $2 = 1+1$)

$$f(x) = \frac{1+x^2+1}{1+x^2} = 1 + \frac{1}{1+x^2}$$

$$\boxed{F(x) = x + \arctan(x) + C.}$$

[4]