Question 1: Determine whether $S = \{4 + 6x + x^2, -1 + 4x + 2x^2, 5 + 2x - x^2\}$ is a basis for P_2.

[5]

Question 2: Find a basis and state the dimension of the subspace V of P_2 consisting of all polynomials $p(x) = a_0 + a_1 x + a_2 x^2$ for which $p(0) = 0$.

[5]

Question 3: Let $\mathbf{u}_1 = (2, -6)$, $\mathbf{u}_2 = (3, 8)$, $\mathbf{w} = (2, 2)$ and $S = \{\mathbf{u}_1, \mathbf{u}_2\}$. Find $(\mathbf{w})_S$. (Recall, $(\mathbf{w})_S$ is the coordinate vector for $\mathbf{w}$ relative to the basis S.)

[5]

Question 4: Find a basis and state the dimension for the solution space of the linear homogeneous system

$$\begin{aligned} 2x_1 + x_2 + 3x_3 + x_4 &= 0 \\ x_1 \qquad + 5x_3 + x_4 &= 0 \\ x_1 + x_2 - 2x_3 \qquad &= 0 \end{aligned}$$

[5]

Question 5: Determine if $\mathbf{b} = \begin{bmatrix} -2 \\ 10 \end{bmatrix}$ is in the column space of $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & -6 \end{bmatrix}$, and if so, express **b** as a linear combination of the columns of **A**.

[5]

Question 6: Find a basis for the subspace of R^4 spanned by $\mathbf{v}_1 = (-1, 1, -2, 1)$, $\mathbf{v}_2 = (3, 3, 6, 1)$ and $\mathbf{v}_3 = (-9, -3, -18, 1)$.

[5]

Question 7: The matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 1 & 2 & 1 & 0 & 2 \\ 3 & 6 & 1 & 0 & 8 \\ -1 & -2 & -2 & 0 & -1 \end{bmatrix}$ has RREF $\mathbf{R} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Give a basis and state the dimension of the row space of $\mathbf{A}$.

[2]

(b) Give a basis and state the dimension of the column space of $\mathbf{A}$.

[2]

(c) Give a basis and state the dimension of the null space of $\mathbf{A}$. (Equivalently, give a basis and state the dimension of the solution space of $\mathbf{Ax} = \mathbf{0}$.)

[3]

(d) Determine rank($\mathbf{A}$) and nullity($\mathbf{A}$) .

[1]

(e) Determine rank($\mathbf{A}^T$) and nullity($\mathbf{A}^T$) .

[1]

Question 8: Let the transformation T_1 represent reflection about the yz-plane:

$$T_1\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} -a \\ b \\ c \end{bmatrix},$$

and T_2 be the transformation which rotates a vector in R^3 counter-clockwise about the z-axis by an angle θ.

(a) Determine the standard matrix **A** for the transformation T_1 .

[3]

(b) Determine the standard matrix **B** for the transformation T_2 .

[3]

(c) Determine the image of the vector $(1, 2, -1)$ if it is first reflected about the yz-plane and then rotated about the z-axis by $\pi/4$ (or 45°.)

[4]