Question 1: Determine an equation of the plane containing the three points $P_1(3,0,2)$, $P_2(4,3,0)$ and $P_3(8,1,-1)$. (You may state your equation of the plane in any form you wish.)

[5]

Question 2: Determine the distance from the line $\mathbf{r} = (1,3,2) + t(1,2,-1)$ to the plane $y + 2z = 5$.

[5]

Question 3: Find vector and parametric equations of the plane through the origin that is orthogonal to $\mathbf{v} = (3, 1, -6)$.

[5]

Question 4: Determine all values of a for which $\mathbf{u} = (-1, 2, 4)$, $\mathbf{v} = (a, 1, 1)$ and $\mathbf{w} = (0, a, 2)$ lie in the same plane.

[5]

Question 5: Suppose $\mathbf{u}\bullet(\mathbf{v}\times\mathbf{w}) = 5$. Determine

$$\mathbf{v}\bullet(\mathbf{w}\times\mathbf{u}) - \mathbf{w}\bullet(\mathbf{u}\times\mathbf{v})$$

[5]

Question 6: Express (if possible) $\mathbf{b} = (7, 8, 9)$ as a linear combination of $\mathbf{u} = (2, 1, 4)$, $\mathbf{v} = (1, -1, 3)$ and $\mathbf{w} = (3, 2, 5)$.

[5]

Question 7: Determine the values of b for which the solution space of following system is a line through the origin:

$$x_1 + x_2 + bx_3 = 0$$
$$x_1 + bx_2 + x_3 = 0$$
$$bx_1 + x_2 + x_3 = 0$$

(notice: $b \neq 1$, otherwise all three equations become $x_1 + x_2 + x_3 = 0$, which is not a line.)

[5]

Question 8: Determine whether the functions $f(x) = x$, $g(x) = \sin(2x)$, $h(x) = \cos(2x)$ are linearly independent.

[5]

Question 9: Let $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ where

$$\mathbf{p}_1 = 2 + x + 4x^2, \quad \mathbf{p}_2 = 1 - x + 3x^2, \quad \mathbf{p}_3 = 3 + 2x + 5x^2$$

Determine whether $\text{span}(S) = P_2$.

[5]

Question 10: Determine if $S = \{(0,0,2,2),(3,3,0,0),(1,1,0,-1)\}$ is linearly dependent or independent. (Hint: this can be done in several different ways, at least two of which are very short.)

[5]