Question 1:

(a) Determine all values of x for which $\mathbf{A} = \begin{bmatrix} x & 0 & 0 \\ x^2 & (x^2-1) & 0 \\ x^3 & x^4 & (x^2+1) \end{bmatrix}$ is invertible.

[3]

(b) Determine $\mathbf{B}^{-1}$ if $\mathbf{B} = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & abc \end{bmatrix}$.

[3]

(c) Let $\mathbf{C} = \begin{bmatrix} 4 & 3 & 2 & x \\ 0 & x & 3 & x \\ 0 & 0 & 1/x & x \\ 0 & 0 & 0 & x/2 \end{bmatrix}$. Determine $\det(\mathbf{C}^{-1})$.

[4]

Question 2: Compute the determinants of the following matrices. Show work or provide explanation to support your answers:

(a) $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

[2]

(b) $\mathbf{B} = \begin{bmatrix} 2 & 1 & 6 & 2 \\ 3 & -2 & 4 & 1 \\ 2 & 1 & 6 & 2 \\ 3 & 5 & 2 & 4 \end{bmatrix}$

[3]

(c) $\mathbf{C} = \begin{bmatrix} -2 & 1 & 4 & -1 \\ 1 & 0 & -1 & 2 \\ 5 & -1 & 2 & 1 \\ 0 & 0 & 3 & -1 \end{bmatrix}$

[5]

Question 3: Suppose $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$. Determine

$$\begin{vmatrix} 2d & 2e & 2f \\ a & b & c \\ g+3a & h+3b & i+3c \end{vmatrix}$$

[4]

Question 4: Let $\mathbf{A}$ be a 3×3 matrix each of whose entries is 1 or 0. What is the largest possible value for $\det(\mathbf{A})$?

[4]

Question 5: Use Cramer's rule to determine the value of x_4 in the following system of equations:

$$\begin{aligned} -x_1 + x_3 &= 0 \\ x_2 - 5x_3 + 3x_4 &= 1 \\ x_2 + 3x_4 &= 0 \\ 2x_2 - 4x_3 + x_4 &= 1 \end{aligned}$$

[7]

Question 6: Suppose $\mathbf{A}$ is 3×3 with $\det(\mathbf{A}) = 4$. Determine $\det\left[(2\mathbf{A})(-3\mathbf{A})^{\mathsf{T}}\right]$.

[3]

Question 7: For this question use the vectors

$$\mathbf{u} = (0, -1, 0), \quad \mathbf{v} = (1, 1, 3), \quad \mathbf{w} = (2, 1, 2)$$

(a) Compute $\|2\mathbf{u} - 3(\mathbf{v} - \mathbf{w})\|$.

[2]

(b) Determine the angle between $\mathbf{v}$ and $\mathbf{w}$ (if giving a calculator answer state units.)

[2]

(c) Find a unit vector pointing in the same direction as $\mathbf{u} - \mathbf{v} + \mathbf{w}$.

[3]

(d) Is it possible to express any vector $\mathbf{a} = (a_1, a_2, a_3)$ as a linear combination of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$? That is, for each vector $\mathbf{a}$ can one always find scalars c_1, c_2 and c_3 so that $\mathbf{a} = c_1\mathbf{u} + c_2\mathbf{v} + c_3\mathbf{w}$?

[3]