In the following A, B and C are matrices while a, b and c are scalars. The sizes of matrices are such that each of the expressions is defined.

Basic Properties

1. $A+B=B+A$
2. $A+(B+C)=(A+B)+C$
3. $A(BC)=(AB)C$
4. $A(B+C)=AB+AC$
5. $(B+C)A=BA+CA$
6. $A(B-C)=AB-AC$
7. $(B-C)A=BA-CA$
8. $a(B+C)=aB+aC$
9. $a(B-C)=aB-aC$
10. $(a+b)C=aC+bC$
11. $(a-b)C=aC-bC$
12. $a(bC)=(ab)C$
13. $a(BC)=(aB)C=B(aC)$

Zero Matrices

Definition: The **zero matrix** of size $m \times n$ is the matrix with each entry 0, denoted $\mathit{0}_{m\times n}$. The size is omitted when the context is clear.

Properties:

1. $A+\mathit{0}=\mathit{0}+A=A$
2. $A-\mathit{0}=A$
3. $A-A=\mathit{0}$
4. $\mathit{0}A=\mathit{0}$
5. If $cA=\mathit{0}$ then $c=0$ or $A=\mathit{0}$

Identity Matrices

Definition: The **identity matrix** of order n is the **square** matrix with main diagonal entries of 1 and zeros elsewhere, denoted I_n. The size is omitted when the context is clear.

Example:

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Properties: Here assume A is $m \times n$:

1. $AI_n = A$
2. $I_m A = A$

Inverse of a Matrix

Definition: If A and B are $n \times n$ with the property that $AB = BA = I$, then B is called the **inverse** of A, written $B = A^{-1}$. That is,

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I$$

For a given square matrix A, if A^{-1} exists A is said to be **invertible** or **nonsingular**. If not invertible, A is said to be **singular**.