Question 1 [10 points]:

(a) Determine $f'(0)$ if $f(x) = \int_0^{(1+x)^2} t^2 e^t \, dt$.

[3]

(b) For this question use the equation

$$6 + \int_a^x \frac{f(t)}{t^2} \, dt = 2\sqrt{x}$$

(i) Determine the value of the constant a .

[2]

(ii) Determine the function f .

[2]

(c) Determine the value of b so that the average value of $f(x) = x^2$ over $[0, b]$ is same as that of $g(x) = x^3$ over $[0, 2b]$.

[3]

Question 2 [10 points]:

(a) Determine $\int 4\sec^2(x) + \frac{\pi}{x^2}\,dx$.

[3]

(b) Determine $\int_1^2 \frac{t^5 - 2t}{t^3}\,dt$.

[3]

(c) A tree's height increases at a rate of $h'(t) = \frac{2}{\sqrt{1+t}} + \frac{3}{1+t}$ meters per year where $t = 0$ corresponds to the present. What is the increase in height during the first three years of growth?

[4]

Question 3 [10 points]: Determine the following integrals:

(a) $\int x \sin(2x^2)\, dx$

[3]

(b) $\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\, dx$

[3]

(c) $\int \sec^3(x)\tan(x)\, dx$

[4]

Question 4 [10 points]: Determine the following integrals:

(a) $\int_0^{\pi} x\cos(x)\,dx$

[5]

(b) $\int (x^2-1)e^x\,dx$

[5]

Question 5 [10 points]: Determine the following integrals:

(a) $\int \sin^2(x)\cos^3(x)\,dx$

[5]

(b) $\int \tan^3(x)\,dx$

[5]