1 Exponentials

General Base
$$a > 0$$

Special Case: Base $e=2.71828\cdots$

$$a^b a^c = a^{b+c}$$

$$e^b e^c = e^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$\frac{e^b}{e^c} = e^{b-c}$$

$$(a^b)^c = a^{bc}$$

$$(e^b)^c = e^{bc}$$

$$\frac{d}{dx}\left[a^{x}\right] = a^{x}\ln\left(a\right)$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

2 Logarithms

Definiton: $\log_a(b)$ is the power to which a is raised to give b.

Definiton: $\ln(b) = \log_e(b)$, the power to which e is raised to give b.

General Base a > 0

Special Case: Base $e=2.71828\cdots$

Laws:

$$\log_a(bc) = \log_a(b) + \log_a(c)$$

$$\ln(bc) = \ln(b) + \ln(c)$$

$$\log_a \left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$$

$$\ln\left(\frac{b}{c}\right) = \ln\left(b\right) - \ln\left(c\right)$$

$$\log_a(b^c) = c \log_a(b)$$

$$\ln\left(b^{c}\right) = c\ln\left(b\right)$$

Change of Base:

$$\log_b(c) = \frac{\log_a(c)}{\log_a(b)}$$

$$\log_b(c) = \frac{\ln(c)}{\ln(b)}$$

Derivative:

$$\frac{d}{dx} \left[\log_a(x) \right] = \frac{1}{x \ln(a)}$$

$$\frac{d}{dx}\left[\ln\left(x\right)\right] = \frac{1}{x}$$

3 Inverse Properties

General Base a > 0

Special Case: Base $e = 2.71828 \cdots$

$$a^{\log_a(x)} = x$$

$$e^{\ln\left(x\right)} = x$$

$$\log_a(a^x) = x$$

$$\ln\left(e^x\right) = x$$