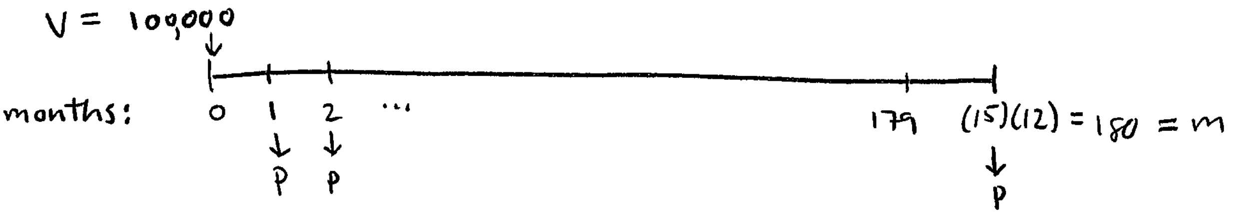
Question 1 [10 points]: Round numerical final answers to two decimal places (i.e. \$123.45, 7.89%, etc.)

(a) A \$100,000 inheritance is placed in a trust fund where it earns 4% interest compounded monthly. Equal payments are made from the fund to the beneficiary at the end of each month. Determine the amount of the monthly payments if the trust fund is to last for 15 years.



$$i = \frac{0.04}{12}.$$

$$V = P\left[\frac{1-(1+i)^{-m}}{i}\right] \Rightarrow P = \frac{iV}{1-(1+i)^{-m}}$$

$$= \frac{(0.04/12)(100{,}000)}{1-\left(1+\frac{0.04}{12}\right)^{-180}}$$

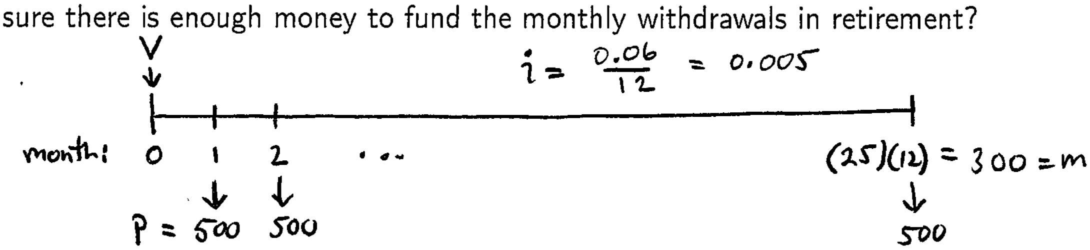
$$= \boxed{\$\,739.69}$$

[5]

(b) On his 45th birthday a person makes a plan to retire on his 65th birthday. He will fund his retirement by making deposits at the end of each month into an account paying 6% interest compounded monthly. When he retires he plans to withdraw \$500 at the end of each month for 25 years. What level of monthly deposits will ensure there is enough money to fund the monthly withdrawals in retirement?

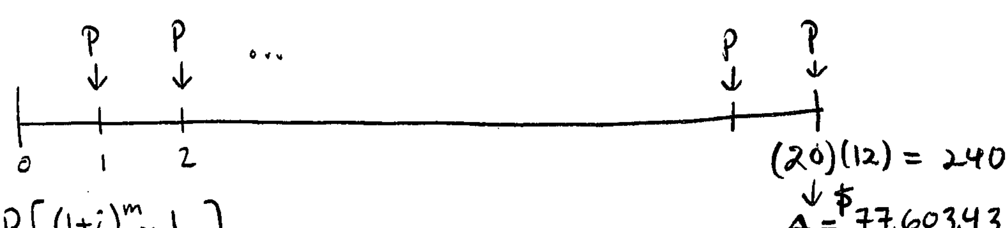
Retirement:

$$i = \frac{0.06}{12} = 0.005$$



$$\therefore V = P\left[\frac{1-(1+i)^{-m}}{i}\right] = 500\left[\frac{1-(1.005)^{-300}}{0.005}\right] = \$77{,}603.43$$

Age 45 → 65:



$$A = P\left[\frac{(1+i)^m-1}{i}\right]$$

$$\therefore P = \frac{iA}{(1+i)^m-1} = \frac{(0.005)(77{,}603.43)}{(1.005)^{240}-1} = \boxed{\$167.96}$$

[5]

Question 2 [10 points]:

(a) For this question use the following sets:

$$U = \{0,1,2,3,4,5,6,7,8,9\}, \quad A = \{0,1,3,5,7\}, \quad B = \{2,4,5,7,9\}, \quad C = \{2,3,4,6,8\}$$

Determine the following:

(i) $\overline{A} \cap \overline{B}$

$\overline{A} = \{2,4,6,8,9\}$

$\overline{B} = \{0,1,3,6,8\}$

$\therefore \overline{A} \cap \overline{B} = \{6,8\}$

[2]

(ii) $A \cup (B \cap C)$

$B \cap C = \{2,4\}$

$\therefore A \cup (B \cap C) = \{0,1,2,3,4,5,7\}$

[2]

(iii) $(B \cup C) \cap \overline{(B \cup A)}$

$B \cup C = \{2,3,4,5,6,7,8,9\}$

$B \cup A = \{0,1,2,3,4,5,7,9\}$

$\overline{B \cup A} = \{6,8\}$

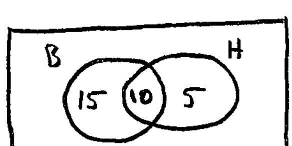
$\therefore (B \cup C) \cap \overline{(B \cup A)} = \{6,8\}$

[3]

(b) A group of people was surveyed and it was found that 25 had brown eyes, 15 had black hair, while 10 people reported having both. If 53 people were surveyed in total, how many had neither brown eyes nor black hair?

B : brown eyes

H : black hair



$n(B) = 25$

$n(H) = 15$

$n(B \cap H) = 10$

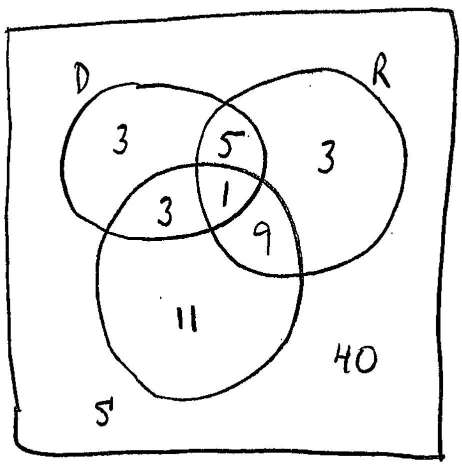
$\therefore n(\overline{B \cup H}) = 53 - (15 + 10 + 5) = \boxed{23}$

[3]

Question 3 [8 points] In a survey of 75 consumers, 12 indicated they were going to buy a new dishwasher, 18 said they were going to buy a new refrigerator, and 24 were going to buy a new stove. Of this 75, 6 said they would buy both a dishwasher and refrigerator, 4 both a dishwasher and stove, and 10 both a stove and refrigerator. Only 1 person was going to buy all three items.

(a) Draw a Venn diagram showing the distribution of purchase options.

Let D: dishwasher, R: refrigerator, S: stove



$75-(3+5+1+3+3+9+11)$

$= 40$

[4]

(b) How many were going to buy none of the items?

$\boxed{40}$

[2]

(c) How many plan to buy a stove and refrigerator but not a dishwasher?

$\boxed{9}$

[2]

Question 4 [10 points]

(a) How many five letter codes are possible using the first ten letters of the alphabet if letters can be repeated within a code but adjacent (that is, side by side) letters cannot be the same?

Choices: 10 9 9 9 9

$\therefore$ number of codes is $(10)(9^4) = \boxed{65{,}610}$

[3]

(b) The 26 lower-case letters of the alphabet and the digits 0, 1, 2, ..., 9 are used to make four character computer passwords. How many passwords are possible if repetition of characters within a password is not allowed?

Choices: 36 35 34 33

$\therefore$ number of passwords is $(36)(35)(34)(33) = \boxed{1{,}413{,}720}$

[3]

(c) Again the 26 lower-case letters of the alphabet and the digits 0, 1, 2, ..., 9 are used to make four character computer passwords. This time, repetition of characters within a password is allowed but passwords must contain at least one letter and at least one digit. How many passwords are possible?

Let p = number of passwords containing at least 1 letter & 1 digit
q = number of passwords without restriction
n = number of passwords without digits (i.e. all letters)
m = number of passwords without letters (i.e. all digits)

Then $p = q - n - m$
$= (36)^4 - (26)^4 - (10)^4$
$= \boxed{1{,}212{,}640}$

[4]

Question 5 [10 points]

(a) A jar contains 3 white, 2 yellow, 4 red and 5 blue marbles. Two marbles are picked at random. What is the probability that exactly 1 is blue?

S = all possible outcomes $\underset{\text{marble 1}}{\underline{\qquad}}$ $\underset{\text{marble 2}}{\underline{\qquad}}$

$n(S) = (14)(13) = 182$

E = exactly one is blue : $\underset{\text{blue}}{\underline{\qquad}}$ $\underset{\text{not blue}}{\underline{\qquad}}$ or $\underset{\text{not blue}}{\underline{\qquad}}$ $\underset{\text{blue}}{\underline{\qquad}}$.

$n(E) = (5)(9) + (9)(5) = 90$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{90}{182} = \boxed{\frac{45}{91} \approx 0.495}$

[3]

(b) Suppose E and F are events with $P(E \cup F) = 5/8$, $P(E \cap F) = 1/3$, and $P(E) = 1/2$. What is the probability that event F does not occur?

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\therefore P(F) = P(E \cup F) + P(E \cap F) - P(E)$

$= \frac{5}{8} + \frac{1}{3} - \frac{1}{2}$

$= \frac{15+8-12}{24}$

$= \frac{11}{24}$

$\therefore P(\bar{F}) = 1 - P(F) = 1 - \frac{11}{24} = \frac{24-11}{24} = \boxed{\frac{13}{24}}$

[3]

(c) In a family of five, what is the probability that at least two have birthdays in the same month?

S = all possible assignments of birth months ; $n(S) = 12^5$

E = "at least two have same birth month"

$\bar{E}$ = "all 5 have different birth months"; $n(\bar{E}) = (12)(11)(10)(9)(8)$

$\therefore P(E) = 1 - P(\bar{E})$

$= 1 - \frac{n(\bar{E})}{n(S)}$

$= 1 - \frac{(12)(11)(10)(9)(8)}{12^5} = \boxed{\frac{89}{144} \approx 0.618}$

[4]