Question 1 [10 points]:

(a) Find the interest I charged and the amount A due if \$400 is borrowed for 9 months at 12% simple interest.

$A = P(1+rt)$, $P = 400$, $t = \frac{9}{12}$, $r = 0.12$

$\therefore \quad A = 400(1+(0.12)(0.75))$

$\boxed{A \doteq \$436}$

$I = A - P$

$= 436 - 400$

$\boxed{I \doteq \$36}$

[2]

(b) \$200 is placed in a savings account paying 4% per annum compounded monthly. How much is in the account after 7 months?

$A = P\left(1+\frac{r}{n}\right)^{nt}$, $P = 200$, $r = 0.04$, $n = 12$, $t = \frac{7}{12}$

$\therefore \quad A = 200\left(1+\frac{0.04}{12}\right)^{(12)\left(\frac{7}{12}\right)}$

$\doteq \boxed{\$204.71}$

[2]

(c) A bank advertises that it pays $3\frac{3}{4}$% interest compounded daily. What is the effective rate of interest?

$r = 0.0375$, $n = 365$

$\therefore \quad R = \left(1+\frac{r}{n}\right)^{n} - 1$

$= \left(1+\frac{0.0375}{365}\right)^{365} - 1$

$\doteq \boxed{3.82\%}$

[3]

(d) How much should be invested today at 5% compounded continuously in order to have \$637 in three years time?

$A = Pe^{rt}$, $A = 637$, $t = 3$, $r = 0.05$

$\therefore \quad P = \frac{A}{e^{rt}} = \frac{637}{e^{(0.05)(3)}} \doteq \boxed{\$548.27}$

[3]

Question 2 [10 points]:

(a) How long will it take money invested at 8% compounded quarterly to double in value?

$$A = P\left(1+\frac{r}{n}\right)^{nt} \quad \text{where } A = 2P, \; r = 0.08, \; n = 4$$

$$2\cancel{P} = \cancel{P}\left(1+\frac{r}{n}\right)^{nt}$$

$$\log_{10}(2) = nt \log_{10}\left(1+\frac{r}{n}\right)$$

$$\Rightarrow t = \frac{\log_{10}(2)}{n\log_{10}\left(1+\frac{r}{n}\right)} = \frac{\log_{10}(2)}{4\log_{10}\left(1+\frac{0.08}{4}\right)} \doteq \boxed{8.75 \text{ yrs}}$$

[3]

(b) At what interest rate compounded semi-annually will money double in 10 years?

$$A = P\left(1+\frac{r}{n}\right)^{nt}, \quad A = 2P, \; r = ?, \; n = 2, \; t = 10$$

$$2\cancel{P} = \cancel{P}\left(1+\frac{r}{2}\right)^{(2)(10)}$$

$$2 = \left(1+\frac{r}{2}\right)^{20}$$

$$2^{1/20} = 1+\frac{r}{2}$$

$$r = \left(2^{1/20}-1\right)(2)$$

$$\boxed{r \doteq 7.05\,\%}$$

[3]

(c) Which is the better loan:

(i) \$5000 for 4 years at 12% simple interest, or

(ii) \$5000 for 4 years at 10% compounded monthly?

(i) $A_1 = P(1+rt)$, $P = 5000$, $r = 0.12$, $t = 4$

$$\therefore A_1 = 5000\left(1+(0.12)(4)\right) = \$7400$$

(ii) $A_2 = P\left(1+\frac{r}{n}\right)^{nt}$, $P = 5000$, $r = 0.1$, $n = 12$, $t = 4$

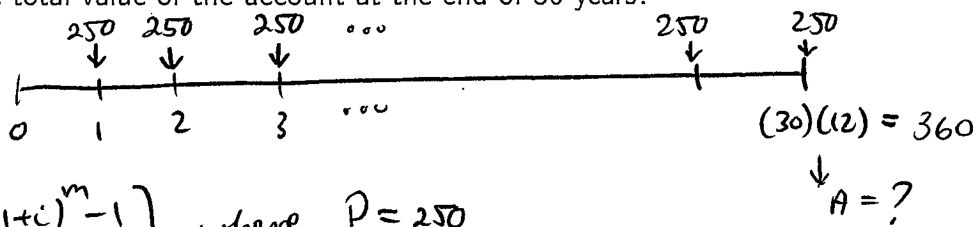
$$\therefore A_2 = 5000\left(1+\frac{0.10}{12}\right)^{(12)(4)} \doteq \$7446.77$$

$\boxed{\therefore \text{ Loan (i) is better.}}$

[4]

Question 3 [10 points]

(a) \$250 is deposited at the end of each month into an account earning 6.5% interest compounded monthly. What is the total value of the account at the end of 30 years?

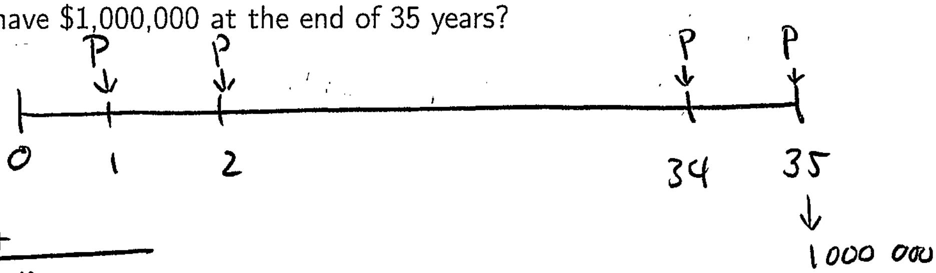


$$A = P\left[\frac{(1+i)^m - 1}{i}\right] \quad \text{where} \quad P = 250, \quad i = \frac{0.0625}{12}, \quad m = 360$$

$$\therefore \; A = 250\left[\frac{\left(1+\frac{0.065}{12}\right)^{360} - 1}{\left(\frac{0.0625}{12}\right)}\right] \doteq \boxed{\$263,479.99}$$

[3]

(b) How much must be deposited at the end of each year into an account earning 9% compounded annually in order to have \$1,000,000 at the end of 35 years?



$$\left.\begin{aligned} i &= 0.09 \\ m &= 35 \end{aligned}\right\}$$

$$P = \frac{iA}{(1+i)^m - 1}$$

$$= \frac{(0.09)(1000\,000)}{(1+0.09)^{35} - 1}$$

$$\doteq \boxed{\$4635.84}$$

[3]

(c) How many years will it take to save \$1,000,000 if you place \$1000 per month into an account earning 7% compounded monthly?

$$A = P\left[\frac{(1+i)^m - 1}{i}\right]$$

$$\frac{iA}{P} = (1+i)^m - 1$$

$$\frac{iA}{P} + 1 = (1+i)^m$$

$$m = \frac{\log_{10}\left(\frac{iA}{P} + 1\right)}{\log(1+i)}$$

$$m = \frac{\log_{10}\left(\frac{\left(\frac{0.07}{12}\right)\cdot 1000000}{1000} + 1\right)}{\log_{10}\left(1 + \frac{0.07}{12}\right)}$$

$$m \doteq 330.41 \text{ months.}$$

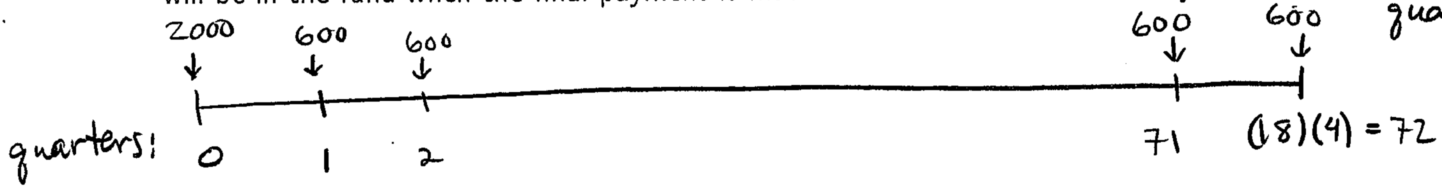
$$\doteq \frac{330.41}{12} \text{ years}$$

$$\doteq \boxed{27.5 \text{ years}}$$

[4]

Question 4 [10 points]

(a) To save for a child's university education, parents make a \$2000 deposit the day their child is born followed by \$600 deposits every three months. If all deposits are made to a fund paying 5% interest compounded quarterly, how much will be in the fund when the final payment is made on the child's 18'th birthday?



Let A_1 = future value of initial \$2000: $P = 2000$, $r = 0.05$, $n=4$, $t=18$

$$\therefore A_1 = P\left(1+\frac{r}{n}\right)^{nt} = 2000\left(1+\frac{0.05}{4}\right)^{(4)(18)} = \$4891.84$$

Let A_2 = amount of the annuity: $P = 600$, $i = \frac{0.05}{4}$, $m = (18)(4) = 72$.

$$\therefore A_2 = P\left[\frac{(1+i)^m - 1}{i}\right] = 600\left[\frac{\left(1+\frac{0.05}{4}\right)^{72} - 1}{\left(\frac{0.05}{4}\right)}\right] \doteq \$69,404.17$$

$\therefore$ Value of fund is $A_1 + A_2 = 4891.84 + 69,404.17 = \boxed{\$74,296.01}$

[5]

(b) Lenny and Carl have a plan to retire in ten years time and they each wish to have \$100,000 saved by retirement day. Lenny starts making deposits at the end of each month into a fund earning 5% interest compounded monthly and continues these deposits for the full ten years. Carl waits three years before starting his monthly deposits, but finds a fund which earns 7% interest compounded monthly. Who has the larger monthly payment, Lenny or Carl?

$$A = P\left[\frac{(1+i)^m - 1}{i}\right] \Rightarrow P = \frac{iA}{(1+i)^m - 1} \quad \text{where } A = 100,000.$$

Lenny: $i = \frac{0.05}{12}$, $m = (12)(10) = 120$.

$$\therefore P_L = \frac{\left(\frac{0.05}{12}\right)(100\,000)}{\left(1+\frac{0.05}{12}\right)^{120} - 1} \doteq \$643.99$$

Carl: $i = \frac{0.07}{12}$, $m = (7)(12) = 84$

$$\therefore P_C = \frac{\left(\frac{0.07}{12}\right)(100\,000)}{\left(1+\frac{0.07}{12}\right)^{84} - 1} \doteq \$925.93$$

$\therefore$ Carl has the larger monthly payment.

[5]

Question 5 [10 points]: A company produces two types of steel. Type 1 requires 2 hours of melting, 4 hours of cutting, and 10 hours of rolling per ton. Type 2 requires 5 hours of melting, 1 hour of cutting, and 5 hours of rolling per ton. Forty hours are available for melting, 20 for cutting, and 60 for rolling. Each ton of Type 1 produces \$240 profit and each ton of Type 2 yields \$80 profit. Determine the maximum profit.

Keep your work organized: define your variables, list all required inequalities, neatly draw any required graphs (use graph paper on the next page) and clearly show your work when determining corner points. State a clear conclusion.

Let x = # tons of Type 1
y = # tons of Type 2.

Maximize profit subject to $z = 240x + 80y$

$2x + 5y \leq 40$ } melting
$4x + y \leq 20$ } cutting
$10x + 5y \leq 60$ } rolling
$x \geq 0$
$y \geq 0$.

Corner Pts:
- By inspection: $(0,0), (5,0), (0,8)$
- Solving: ① $4x+y=20$, ② $10x+5y=60$; ① $\Rightarrow y = 20-4x$; ② $\Rightarrow 10x+5(20-4x)=60$; $-10x = -40$; $x = 4$; $\therefore y = 20-4(4) = 4$. $\therefore (4,4)$

① $10x+5y=60$, ② $2x+5y=40$; ① $\Rightarrow 5y = 60-10x$; ② $\Rightarrow 2x+(60-10x)=40$; $-8x = -20$; $x = \frac{5}{2} = 2.5$; $\therefore y = \frac{60-10x}{5} = \frac{60-10(2.5)}{5} = 7$. $\therefore (2.5,7)$

$\longrightarrow$

[10]

Question 5 (continued):

corner pts	$z = 240x + 80y$
(0,0)	0
(5,0)	1200
(0,8)	640
(4,4)	1280
(2.5,7)	1160

∴ Maximum profit is \$1280