Question 1 [10 points]:

(a) A system of three equations in the variables x, y and z has been partially reduced using Gaussian elimination, resulting in the following matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & -4 & -9 \end{array}\right]$$

Complete the matrix reduction and state the solution to the system. If there is no solution, state "no solution".

[3]

(b) A system of three equations in the variables x and y has been partially reduced using Gaussian elimination, resulting in the following matrix:

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & -3 & -6 \\ 0 & 1 & 2 \end{array}\right]$$

Complete the matrix reduction and state the solution to the system. If there is no solution, state "no solution".

[2]

(c) Solve the following system of equations using Gaussian (or Gauss-Jordan) elimination:

$$\begin{aligned} 2x - 3y + 4z &= 7 \\ x - 2y + 3z &= 2 \end{aligned}$$

[5]

Question 2 [10 points]: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 4 & -3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ -3 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 5 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$$

Calculate the following, if possible. If an operation is not defined, state "not defined":

(a) $\mathbf{A} + 6\mathbf{C}$

[2]

(b) $\mathbf{CB} - 2\mathbf{DI}_2$

[3]

(c) $\mathbf{AC} - \mathbf{BD}$

[2]

(d) Suppose there is some matrix $\mathbf{P}$ such that the product $\mathbf{DCPA}$ is defined. What must be the size of the matrix $\mathbf{P}$?

[3]

Question 3 [10 points]

(a) Determine $\mathbf{A}^{-1}$ where $\mathbf{A}$ is the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$$

[5]

(b) Let $\mathbf{B} = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$. Find $\mathbf{B}^{-1}$ or show that it does not exist.

[2]

(c) Suppose a matrix $\mathbf{C}$ has inverse $\mathbf{C}^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$. Use this to find a matrix $\mathbf{D}$ so that

$$\mathbf{CD} - \mathbf{I}_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

[3]

Question 4 [10 points]: Maximize $z = 3x + 4y$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 2 \\ 3x + 2y &\leq 12 \\ y &\leq 5 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Keep your work organized: neatly draw any required graphs (use graph paper on the next page) and clearly show your work when determining corner points. State a clear conclusion.

[10]

Question 4 (continued):