Question 1. [10]:

(a) Find an equation of the line containing the points $(4, -2)$ and $(-7, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{-7-4} = \frac{-5}{11}$$

$$\therefore \quad y - y_1 = m(x - x_1)$$

$$\boxed{y + 2 = \frac{-5}{11}(x-4)}$$

$$\underline{\text{or}} \quad y = \left(\frac{-5}{11}\right)x - \frac{2}{11}$$

[4]

(b) Find an equation of the horizontal line passing through the point $(-1, 6)$.

$$\boxed{y = 6}$$

[2]

(c) Find both the slope and y-intercept of the line $\frac{1}{4}x - \frac{1}{3}y = \frac{5}{12}$.

$$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{12}$$

$$12\left[\frac{1}{4}x - \frac{1}{3}y\right] = \left[\frac{5}{12}\right]12$$

$$3x - 4y = 5$$

$$-4y = -3x + 5$$

$$y = \frac{3}{4}x - \frac{5}{4}$$

$\therefore\ m = \frac{3}{4}$, y-intercept $\left(0, \frac{-5}{4}\right)$.

[4]

Question 2. [10]:

(a) The following lines are parallel. Determine the value of the number p:

$$\begin{aligned} L: &\quad 2x + 3y = -5 \\ M: &\quad -4x + py = 1 \end{aligned}$$

$L: 2x+3y=-5$

$3y = -2x - 5$

$y = -\frac{2}{3}x - \frac{5}{3}$

$M: -4x+py=1$

$py = 4x+1$

$y = \frac{4}{p}x + \frac{1}{p}$

L & M parallel $\Rightarrow$ slopes are equal

$\Rightarrow -\frac{2}{3} = \frac{4}{p}$

$\Rightarrow p = -\frac{12}{2}$

$\Rightarrow \boxed{p = -6}$

[4]

(b) Determine whether the following lines are parallel, coincident or intersecting. If intersecting, determine the point of intersection:

$$\begin{aligned} L: &\quad x - 4y = 2 \\ M: &\quad 3x + 5y = 0 \end{aligned}$$

$L: x-4y=2$

$-4y = -x+2$

$y = \frac{1}{4}x - \frac{1}{2}$

$M: 3x+5y=0$

$y = -\frac{3}{5}x$

slopes differ, so lines are $\boxed{\text{intersecting}}$

Let $y = -\frac{3}{5}x$ in L: $-\frac{3}{5}x = \frac{1}{4}x - \frac{1}{2}$

$\Rightarrow \left(\frac{1}{4} + \frac{3}{5}\right)x = \frac{1}{2}$

$\Rightarrow \frac{17}{20}x = \frac{1}{2}$

$\Rightarrow x = \frac{10}{17}$

$\therefore y = -\frac{3}{5}\left(\frac{10}{17}\right) = \frac{-6}{17}$

$\boxed{\therefore \text{ lines intersect at } \left(\frac{10}{17}, \frac{-6}{17}\right)}$

[6]

Question 3. [5]: An investor earned \$415 in one year from the investment of \$5,000. Part of the \$5,000 was invested at 8% while the rest earned interest at 9%. How much interest income was earned from the 8% investment?

Let x = amount invested at 8%
y = amount invested at 9%.

① $x + y = 5000$
② $0.08x + 0.09y = 415$

① $\Rightarrow$ $y = 5000 - x$

② $\Rightarrow$ $0.08x + 0.09(5000 - x) = 415$

$0.08x + 450 - 0.09x = 415$

$-0.01x = 415 - 450$

$x = \frac{-35}{-0.01} = 3500$

$\therefore (0.08)(3500) = \$280.$

$\therefore$ \$280 was earned from the 8% investment

[5]

Question 4. [5]: A company produces watches and sells them for \$19.50 a piece. Each watch costs \$12.30 to produce. During a one week period 1840 watches were produced at a total production cost of \$25,260. Assuming the fixed cost and the cost to produce each watch remain the same each week, determine the number of watches that must be produced to break-even on a weekly basis.

Let x = number of watches produced in a week,
a = fixed cost.

Cost: $C = a + 12.30x$

revenue: $R = 19.50x$.

When $x = 1840$, $C = \$25,260$, so $25,260 = a + (12.30)(1840)$

$\Rightarrow$ $a = 25,260 - (12.30)(1840)$
$a = 2628$

$\therefore C = 2628 + 12.30x$.

At break even $R = C$, so

$19.50x = 2628 + 12.30x$

$\Rightarrow x = \frac{2628}{(19.50 - 12.30)} = 365$

$\therefore$ 365 watches must be produced

[5]

Question 5. [5]: A certain product has supply equation $S = (2/3)p$ where p is price in dollars and S is the quantity supplied. At equilibrium $p = 54$. When p increases to 57 the quantity demanded is 32. Determine the demand equation.

At $p = 54$, $S = \frac{2}{3}(54) = 36$, so $(54, 36)$ is on S-line, and $(54, 36)$ is also on D-line since $p = 54$ is equilibrium price.

Also, $(57, 32)$ is on D-line.

$\therefore m = \frac{36-32}{54-57} = \frac{-4}{3}$

$\therefore$ D-line has equation $D - 32 = \frac{-4}{3}(p - 57)$

or $D - 36 = \frac{-4}{3}(p - 54)$

or $D = \frac{-4}{3}p + 108$

[5]

Question 6. [5]: The average expected lifespan of a newborn female was 78.8 years in 1990. By 2010 that number had risen to 81.4. Assuming the relationship between year of birth and expected lifespan is linear, in what year will expected lifespan be 84?

We have

x	y
1990	78.8
2010	81.4

$m = \frac{81.4 - 78.8}{2010 - 1990} = 0.13.$

$\therefore y - 81.4 = 0.13(x - 2010)$

Letting $y = 84$: $84 - 81.4 = 0.13(x - 2010)$

$\therefore x = \frac{84 - 81.4}{0.13} + 2010 = 2030.$

$\therefore$ Expected lifespan will be 84 in 2030.

[5]

Question 7 [10]: Solve the following system of equations **using matrix reduction: Gaussian or Gaus-Jordan elimination** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$2x + 3y - z = -2$$
$$x - y + z = 8$$
$$3x - 2y - 9z = 9$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & -2 \\ 1 & -1 & 1 & 8 \\ 3 & -2 & -9 & 9 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 2 & 3 & -1 & -2 \\ 3 & -2 & -9 & 9 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$:
$R_3 = (-3)r_1 + r_3$: $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 5 & -3 & -18 \\ 0 & 1 & -12 & -15 \end{array}\right]$$

$r_2 \leftrightarrow r_3$: $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 5 & -3 & -18 \end{array}\right]$$

$R_3 = (-5)r_2 + r_3$;

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & -57 & 57 \end{array}\right]$$

$R_3 = \left(\frac{1}{57}\right)r_3$: $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore z = 1$

$y = -15 + 12z = -15 + 12 = -3$

$x = 8 + y - z = 8 - 3 - 1 = 4$

$\therefore (x, y, z) = (4, -3, 1).$

[10]