Question 1 [10]: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to state the row operations used at each step and clearly state the final solution.

$$x - 2y - z - 3w = -3$$
$$-x + y + z = 2$$
$$4y + 3z - 6w = -2$$

Question 2 [10]: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to state the row operations used at each step and clearly state the final solution.

$$
\begin{aligned}
-2x + 6y - z &= -10 \\
x - 2y + 4z &= 6 \\
x + y + 13z &= 6
\end{aligned}
$$

Question 3: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & -4 \\ 1 & 0 \\ 2 & -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

(a)[3] Compute $(3\mathbf{A} - 4\mathbf{C})\mathbf{D}$

(b)[3] Compute $(\mathbf{AB} + 3\mathbf{I}_3)^T$

(c)[2] Compute $\text{tr}(\mathbf{AB} - \mathbf{BA})$

(d)[2] Suppose there is some matrix **P** such that the product **APC** is defined. What must be the size of the matrix **P**?

Question 4:

(a)[7] Determine $\mathbf{A}^{-1}$ where $\mathbf{A}$ is the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

(b)[3] Write the following system as a matrix product $\mathbf{Ax} = \mathbf{b}$ and use your result in part (a) to solve the system:

$$\begin{aligned} x - y &= 3 \\ -x + 2y + 3z &= 1 \\ x + 2z &= -7 \end{aligned}$$

Question 5:

(a)[7] Determine all values of c for which the following matrix is invertible:

$$\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$$

(b)[3] Find two non-zero matrices **A** and **B** such that $\mathbf{AB} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.