(1) Determine if the following vectors form a basis for R^3: $\{(3,1,-4),(2,5,6),(1,4,8)\}$.

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{vmatrix} = 3(16) - 2(24) + 1(26)$$

$$= 26$$

$$\neq 0.$$

$\therefore$ The vectors are linearly independent and span R^3, so are a basis.

[4]

(2) Let $\mathbf{v} = (2,-1,3)$ and $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ where $\mathbf{v}_1 = (1,0,0)$, $\mathbf{v}_2 = (2,2,0)$ and $\mathbf{v}_3 = (3,3,3)$. Determine $(\mathbf{v})_S$, the coordinates of $\mathbf{v}$ relative to the basis S.

Want $a\vec{v}_1 + b\vec{v}_2 + c\cdot\vec{v}_3 = \vec{v}$

$$\Rightarrow a\begin{bmatrix}1\\0\\0\end{bmatrix} + b\begin{bmatrix}2\\2\\0\end{bmatrix} + c\begin{bmatrix}3\\3\\3\end{bmatrix} = \begin{bmatrix}2\\-1\\3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

$$\Rightarrow c = 1, \quad b = \frac{-1-3(1)}{2} = -2, \quad a = 2-3(1)-2(-2) = 3$$

$\therefore (\vec{v})_S = (3,-2,1).$

[5]

(3) Find a basis for the following subspaces of R^3:

(i) The plane $3x - 2y + 5z = 0$ $\Rightarrow$ $x = \frac{2}{3}y - \frac{5}{3}z$

$\therefore$ solutions are $y = r$, $z = t$, $x = \frac{2}{3}r - \frac{5}{3}t$

$\Rightarrow \left(\frac{2}{3}r - \frac{5}{3}t, r, t\right)$

$= r\left(\frac{2}{3}, 1, 0\right) + t\left(-\frac{5}{3}, 0, 1\right)$

$\therefore$ Basis is $\left\{\left(\frac{2}{3}, 1, 0\right), \left(-\frac{5}{3}, 0, 1\right)\right\}$

[3]

(ii) The line $x = 2t$, $y = -t$, $z = 4t$

Solutions are $t(2, -1, 4)$

$\therefore$ Basis is $\{(2, -1, 4)\}$.

[3]