 Name: SOLUTIONS.

(1) [4] Find the area of the triangle in 3-space that has vertices $P_1(2,6,-1)$, $P_2(1,1,1)$ and $P_3(4,6,2)$.

$\vec{u} = \overrightarrow{P_1P_2} = (-1,-5,2)$

$\vec{v} = \overrightarrow{P_1P_3} = (2,0,3).$

$$A = \frac{1}{2}\|\vec{u}\times\vec{v}\| = \frac{1}{2}\left\| \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -5 & 2 \\ 2 & 0 & 3 \end{bmatrix} \right\|$$

$$= \frac{1}{2}\|-15\hat{i} + 7\hat{j} + 10\hat{k}\|$$

$$= \frac{1}{2}\sqrt{15^2+7^2+10^2}$$

$$= \frac{1}{2}\sqrt{374}$$

(2) [4] Determine whether the set of all polynomials $a_0+a_1x+a_2x^2+a_3x^3$ for which $a_0+a_1+a_2+a_3=0$ is a subspace of P_3.

Let W be the set in question, and suppose $\vec{u} = a_0+a_1x+a_2x^2+a_3x^3$ and $\vec{v} = b_0+b_1x+b_2x^2+b_3x^3$ are both in W.

If r and t are any scalars, then

$$r\vec{u}+t\vec{v} = (ra_0+tb_0)+(ra_1+tb_1)x+(ra_2+tb_2)x^2+(ra_3+tb_3)x^3,$$

and $r\vec{u}+t\vec{v}$ is again in W since

$$(ra_0+tb_0)+(ra_1+tb_1)+(ra_2+tb_2)+(ra_3+tb_3)$$
$$= r(a_0+a_1+a_2+a_3)+t(b_0+b_1+b_2+b_3)$$
$$= r(0)+b(0)$$
$$= 0.$$

$\boxed{\therefore W \text{ is a subspace of } P_3}$

(3) [4] Let $S = \{(2,-1,3),(4,1,2),(8,-1,8)\}$. Determine if span$(S) = R^3$.

$\text{Span}(S) = R^3 \iff x(2,-1,3) + y(4,1,2) + z(8,-1,8) = \vec{b}$
has solutions for every $\vec{b}$ in R^3.

Write this system as $\begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{b}$. $\Big\} *$

Since $\det\begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix} = 2(10) - 4(-5) + 8(-5) = 0$,
the system $*$ does not have solutions for every $\vec{b}$ in R^3,
so $\text{Span}(S) \neq R^3$.

(4) [3] Let $\mathbf{v}_1 = (-6,7,2)$, $\mathbf{v}_2 = (3,2,4)$ and $\mathbf{v}_3 = (4,-1,2)$ be vectors with initial points at the origin. Determine whether the three vectors are linearly dependent (equivalently, determine whether all three lie in the same plane.)

Suppose $a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3 = 0$

$\Rightarrow \begin{bmatrix} -6 & 3 & 4 \\ 7 & 2 & -1 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \vec{0}$. $\Big\} *$

$\det\begin{bmatrix} -6 & 3 & 4 \\ 7 & 2 & -1 \\ 2 & 4 & 2 \end{bmatrix} = -6(8) - 3(16) + 4(24) = 0$

So $(*)$ has non-trivial solutions $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

So $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are <u>dependent</u>.