 Name: SOLUTIONS.

(1) [5] Let $\mathbf{u} = (1, -1, 3, 5)$ and $\mathbf{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\mathbf{u}+b\mathbf{v} = (1, -4, 9, 18)$

$$a\vec{u} + b\vec{v} = (a+2b, -a+b, 3a, 5a-3b) = (1, -4, 9, 18)$$

$$\Rightarrow \begin{array}{ll} ① & a+2b = 1 \\ ② & -a+b = -4 \\ ③ & 3a = 9 \\ ④ & 5a-3b = 18 \end{array}$$

③ $\Rightarrow a = 3$, and ② $\Rightarrow b = -4 + a = -1$

and $a = 3$, $b = -1$ also satisfy ① and ④.

$\therefore \boxed{a = 3, \ b = -1}$

(2) [3] Let $\mathbf{v} = (-2, 3, 0, 6)$. Find all scalars k such that $\|k\mathbf{v}\| = 5$.

$$\|k\vec{v}\| = 5$$

$$\Rightarrow |k| \, \|\vec{v}\| = 5$$

$$\Rightarrow |k| = \frac{5}{\|\vec{v}\|}$$

$$\Rightarrow k = \pm \frac{5}{\|\vec{v}\|} = \pm \frac{5}{\sqrt{4+9+36}}$$

$$= \boxed{\pm \frac{5}{7}}$$

(3) [4] Let $\mathbf{u} = (3,3,3)$ and $\mathbf{v} = (1,0,4)$.

1. Determine $\|\mathbf{u}-\mathbf{v}\|$ $= \sqrt{(3-1)^2 + (3-0)^2 + (3-4)^2}$

$$= \sqrt{4+9+1}$$
$$= \boxed{\sqrt{14}}$$

2. Determine the angle θ between $\mathbf{u}$ and $\mathbf{v}$ (report your answer in degrees, rounded to 1 decimal.)

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\Rightarrow \quad \theta = \cos^{-1}\left[\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\| \|\vec{v}\|}\right]$$

$$= \cos^{-1}\left[\frac{(3)(1) + (3)(0) + (3)(4)}{\sqrt{9+9+9}\sqrt{1+0+16}}\right]$$

$$\approx \boxed{45.6^\circ}$$

(4) [3] Find a unit vector that is orthogonal to both $\mathbf{u} = (1,0,1)$ and $\mathbf{v} = (0,1,1)$.

Let $\vec{\omega} = (a,b,c)$

$$\left.\begin{aligned} \vec{\omega}\cdot\vec{u} = 0 &\Rightarrow a + c = 0 \\ \vec{\omega}\cdot\vec{v} = 0 &\Rightarrow b + c = 0 \end{aligned}\right\} \begin{aligned} a &= -c \\ b &= -c \end{aligned}$$

$\therefore$ $\vec{\omega} = (-c,-c,c)$ is orthogonal to both $\vec{u}$ & $\vec{v}$.

Select $c=1$ to get $\vec{\omega} = (-1,-1,1)$.

Now normalize: $\vec{n} = \dfrac{\vec{\omega}}{\|\vec{\omega}\|} = \dfrac{(-1,-1,1)}{\sqrt{3}} = \boxed{\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$