(1) [5] Compute det($\mathbf{A}$) where

$$\mathbf{A} = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

$R_3 = r_3 + r_4$:

$$B = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 6 & 11 & 0 & 2 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

expanding down column 3.

$$\therefore \det(A) = \det(B) = -3 \left|\begin{array}{ccc} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 6 & 11 & 2 \end{array}\right| \begin{array}{cc} 3 & 3 \\ 2 & 2 \\ 6 & 11 \end{array}$$

$$= -3\left[(12 - 36 + 110) - (60 - 66 + 12)\right]$$

$$= \boxed{-240}$$

(2) [3] Suppose $\underbrace{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}_{A} = -4$. Determine $\underbrace{\begin{vmatrix} -2a & -2b & -2c \\ d & e & f \\ g+3d & h+3e & i+3f \end{vmatrix}}_{B}$.

B is obtained from A by $R_1 = -2r_1$ and $R_3 = 3r_2 + r_3$.
The first scales $\det(A)$ by -2, the second row operation has no effect on determinants.

$$\therefore \det(B) = -2\det(A) = (-2)(-4) = \boxed{8}$$

(3) [5] Use Cramer's rule to solve for z:

$$4x + 5y = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}$; $\det(A) = (0)(\sim) - 2(15) + 2(-51)$

$= -132$

$A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}$; $\det(A_3) = 4(-14) - 5(8) + 2(54)$

$= 12$

$\therefore z = \dfrac{\det(A_3)}{\det(A)} = \dfrac{12}{-132} = \boxed{-\dfrac{1}{11}}$

(4) [2] If $\mathbf{A}$ is an $n \times n$ matrix with $\det(\mathbf{A}) = 7$ what is $\det\left((2\mathbf{A})^{-1}\right)$?

$\det((2A)^{-1}) = \dfrac{1}{\det(2A)}$

$= \dfrac{1}{2^n \det(A)}$

$= \boxed{\dfrac{1}{2^n \cdot 7}}$