

Let

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

(1) [5] Compute $BC - 3A^T$ if possible. (If the expression is not defined then say so.)

$$\begin{aligned} BC - 3A^T &= \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} - 3 \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix} - \begin{bmatrix} 9 & -3 & 3 \\ 0 & 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -8 & 18 & 0 \\ 6 & -4 & 7 \end{bmatrix}. \end{aligned}$$

(2) [5] Compute $\text{tr}(ABC)$ if possible. (If the expression is not defined then say so.)

$$\begin{aligned} ABC &= \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix} \end{aligned}$$

$\therefore \text{tr}(ABC)$
 $= 3 + (-11) + 13$
 $= \boxed{5}$

(3) [5] Find all values of k so that

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} k+1 & k+2 & -1 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow k(k+1) + k+2 - 1 = 0$$

$$k^2 + k + k + 2 - 1 = 0$$

$$k^2 + 2k + 1 = 0$$

$$(k+1)^2 = 0$$

$$\boxed{k = -1}.$$