Let

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

(1) [5] Compute $\mathbf{AB} - 2\mathbf{C}^T$ if possible. (If the expression is not defined then say so.)

$$AB - 2C^T = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 8 & 2 \\ 4 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -9 \\ -12 & 3 \\ 0 & -9 \end{bmatrix}$$

(2) [5] Compute $\text{tr}(\mathbf{BCA})$ if possible. (If the expression is not defined then say so.)

$$BCA = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 33 \\ 26 & 14 \end{bmatrix}$$

$$\therefore \quad tr(BCA) = -9 + 14 = \boxed{5}$$

(3) [5] Find all values of k so that

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} k+1 & k+2 & -1 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad k(k+1) + k + 2 - 1 = 0$$

$$k^2 + k + k + 2 - 1 = 0$$

$$k^2 + 2k + 1 = 0$$

$$(k+1)^2 = 0$$

$$\boxed{k = -1}$$