(1) [15] Use Gauss-Jordan elimination to solve the following system of equations:

$$\begin{aligned} 3x - 7y + 4z &= 10 \\ x + 2y - 3z &= -1 \\ -x - y - 2z &= -8 \end{aligned}$$

Clearly state the row operations used at each step of the reduction, and state a clear conclusion.

$$\begin{bmatrix} 3 & -7 & 4 & 10 \\ 1 & 2 & -3 & -1 \\ -1 & -1 & -2 & -8 \end{bmatrix}$$

$r_1 \leftrightarrow r_2$:

$$\begin{bmatrix} \textcircled{1} & 2 & -3 & -1 \\ 3 & -7 & 4 & 10 \\ -1 & -1 & -2 & -8 \end{bmatrix}$$

$R_2 = (-3)r_1 + r_2$:
$R_3 = r_1 + r_3$:

$$\begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & -13 & 13 & 13 \\ 0 & 1 & -5 & -9 \end{bmatrix}$$

$R_2 = \left(\frac{-1}{13}\right) r_2$:

$$\begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & \textcircled{1} & -1 & -1 \\ 0 & 1 & -5 & -9 \end{bmatrix}$$

$R_3 = (-1)r_2 + r_3$:

$$\begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -4 & -8 \end{bmatrix}$$

$R_3 = \left(-\frac{1}{4}\right) r_3$:

$$\begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & \textcircled{1} & 2 \end{bmatrix}$$

$R_1 = 3r_3 + r_1$:
$R_2 = r_3 + r_2$:

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 = (-2)r_2 + r_1$:

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\therefore x = 3$, $y = 1$, $z = 2$