

(1) [7] Evaluate $\int_0^{\pi/4} \sec^4(x) \tan^4(x) dx$

$$\begin{aligned} I &= \int_0^{\pi/4} \sec^4(x) \tan^4(x) dx \\ &= \int_0^{\pi/4} \sec^2(x) \tan^4(x) \sec^2(x) dx \\ &= \int_0^{\pi/4} (1 + \tan^2(x)) \tan^4(x) \sec^2(x) dx \end{aligned}$$

$$\begin{aligned} \text{let } u = \tan(x) & \left\{ \begin{array}{l} x=0 \Rightarrow u=0 \\ x=\frac{\pi}{4} \Rightarrow u=\tan\left(\frac{\pi}{4}\right)=1 \end{array} \right. \\ du = \sec^2(x) dx & \end{aligned}$$

$$\begin{aligned} \therefore I &= \int_0^1 (1+u^2) u^4 du \\ &= \int_0^1 (u^4 + u^6) du \\ &= \left[\frac{u^5}{5} + \frac{u^7}{7} \right]_0^1 \\ &= \frac{1}{5} + \frac{1}{7} \\ &= \boxed{\frac{12}{35}} \end{aligned}$$

(2) [8] Determine $\int \frac{1}{x\sqrt{9-x^2}} dx$

$$I = \int \frac{1}{x\sqrt{9-x^2}} dx \quad \text{let } x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\therefore I = \int \frac{1}{3 \sin \theta \sqrt{9-9 \sin^2 \theta}} \cdot 3 \cos \theta d\theta$$

$$= \frac{1}{3} \int \frac{1}{\sin \theta \cancel{\cos \theta}} \cdot \cancel{\cos \theta} d\theta$$

$$= \frac{1}{3} \int \csc \theta d\theta$$

$$= \frac{1}{3} \ln | \csc \theta - \cot \theta | + C$$

$$= \frac{1}{3} \ln \left| \frac{3}{x} - \frac{\sqrt{9-x^2}}{x} \right| + C$$

