(1) [7] Evaluate $\int_0^{\pi/4} \sec^4(x)\tan^4(x)\,dx$

$$I = \int_0^{\frac{\pi}{4}} \sec^4(x)\tan^4(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} \sec^2(x)\ \tan^4(x)\ \sec^2(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} (1+\tan^2(x))\tan^4(x)\sec^2(x)\,dx$$

let $u = \tan(x)$, $du = \sec^2(x)\,dx$ $\Big\}$ $x=0 \Rightarrow u=0$, $x=\frac{\pi}{4} \Rightarrow u=\tan(\frac{\pi}{4})=1$

$$\therefore\ I = \int_0^1 (1+u^2)u^4\,du$$

$$= \int_0^1 (u^4+u^6)\,du$$

$$= \left[\frac{u^5}{5}+\frac{u^7}{7}\right]_0^1$$

$$= \frac{1}{5}+\frac{1}{7}$$

$$= \boxed{\frac{12}{35}}$$

(2) [8] Determine $\int \frac{1}{x\sqrt{9-x^2}}\,dx$

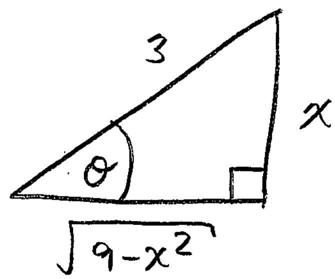
$$I = \int \frac{1}{x\sqrt{9-x^2}}\,dx \qquad \text{let } x = 3\sin\theta$$
$$dx = 3\cos\theta\,d\theta$$

$$\therefore I = \int \frac{1}{\cancel{3}\sin\theta\sqrt{9-9\sin^2\theta}}\cdot\cancel{3}\cos\theta\,d\theta$$

$$= \frac{1}{3}\int \frac{1}{\sin\theta\,\cancel{\cos\theta}}\cdot\cancel{\cos\theta}\,d\theta$$

$$= \frac{1}{3}\int \csc\theta\,d\theta$$

$$= \frac{1}{3}\ln\left|\csc\theta - \cot\theta\right| + C$$



$$= \boxed{\frac{1}{3}\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + C}$$