 Name: SOLUTIONS.

(1) [2] Compute $h'(x)$ where $h(x) = \int_0^{x^3} \ln(1+r^2)\,dr$.

$$h'(x) = \ln(1+(x^3)^2)\,(3x^2)$$

$$= \boxed{\ln(1+x^6)\,(3x^2)}$$

(2) [3] Determine $\int x^3 e^{x^4}\,dx$.

$u = x^4$

$du = 4x^3 dx$

$$I = \frac{1}{4}\int e^u\,du$$

$$= \frac{1}{4}e^u + C$$

$$= \boxed{\frac{1}{4}e^{x^4} + C}$$

(3) [5] Evaluate $\displaystyle\int_0^{\pi/2} \sin(x)\sin(\cos(x))\,dx$.

let $u = \cos(x)$, $du = -\sin(x)\,dx$ $\Big\}$ $x=0 \Rightarrow u = \cos(0) = 1$, $x = \frac{\pi}{2} \Rightarrow u = \cos\left(\frac{\pi}{2}\right) = 0$

$$\therefore\ I = -\int_1^0 \sin(u)\,du$$

$$= \int_0^1 \sin(u)\,du$$

$$= -\cos(u)\Big|_0^1$$

$$= -\cos(1) - (-\cos(0))$$

$$= \boxed{1 - \cos(1)}$$

(4) [5] Determine $\displaystyle\int t^5 \ln t\,dt$.

$u = \ln(t)$ $\quad dv = t^5$

$du = \frac{1}{t}dt$ $\quad v = \frac{t^6}{6}$

$$I = \int u\,dv = uv - \int v\,du$$

$$= \ln(t)\frac{t^6}{6} - \int \frac{t^6}{6}\frac{1}{t}\,dt$$

$$= \ln(t)\cdot\frac{t^6}{6} - \frac{1}{6}\int t^5\,dt$$

$$= \boxed{\ln(t)\frac{t^6}{6} - \frac{1}{36}t^6 + C}$$