 Name: SOLUTIONS.

(1) [5] Evaluate $\lim\limits_{x\to 0^+} \dfrac{x^2}{x - \sin(x)}$. $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0^+} \frac{2x}{1-\cos(x)} \sim \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{2}{\sin(x)}$$

$$= \boxed{\infty}$$

(2) [5] Determine $f(x)$ if

$$f''(x) = 3x + \cos(x), \quad f(0) = 0, \quad f'(0) = 2$$

$$f'(x) = \frac{3x^2}{2} + \sin(x) + C_1$$

$$f'(0) = 2 \Rightarrow \quad 2 = \tfrac{3}{2}(0)^2 + \sin(0) + C_1$$

$$\therefore \; C_1 = 2$$

$$\therefore \; f'(x) = \tfrac{3}{2}x^2 + \sin(x) + 2.$$

$$f(x) = \tfrac{1}{2}x^3 - \cos(x) + 2x + C_2$$

$$f(0) = 0 \Rightarrow \quad 0 = \tfrac{1}{2}(0)^3 - \cos(0) + 2(0) + C_2$$

$$\therefore \; C_2 = 1$$

$$\therefore \; \boxed{f(x) = \tfrac{1}{2}x^3 - \cos(x) + 2x + 1}$$

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(3) [5] Evaluate $\lim_{x\to 0^+} (\sin(2x))^x$. $\sim 0^0$

$$\sin(2x)^x = e^{x \ln[\sin(2x)]}$$

$$\lim_{x\to 0^+} x \ln[\sin(2x)]$$

$$= \lim_{x\to 0^+} \frac{\ln[\sin(2x)]}{\frac{1}{x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\sin(2x)} \cdot 2\cos(2x)}{\frac{-1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2x^2\cos(2x)}{\sin(2x)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-4x\cos(2x) + 4x^2\sin(2x)}{2\cos(2x)}$$

$$= 0$$

$$\therefore \lim_{x\to 0^+} (\sin(2x))^x = e^0 = \boxed{1}.$$