 Name: SOLUTIONS.

(1) [5] Evaluate $\lim_{x\to 0} \frac{x^2}{1-\cos x}$. $\sim$ "$\frac{0}{0}$"

$$\stackrel{H}{=} \lim_{x\to 0} \frac{2x}{0+\sin(x)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{2}{\cos(x)}$$

$$= 2$$

(2) [5] Evaluate $\lim_{x\to 0^+} (\tan(2x))^x$. $\sim$ "0^0"

$$(\tan(2x))^x = e^{x\ln[\tan(2x)]}$$

$$\lim_{x\to 0^+} x\ln[\tan(2x)]$$

$$= \lim_{x\to 0^+} \frac{\ln[\tan(2x)]}{\frac{1}{x}} \sim \text{"}-\frac{\infty}{\infty}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\tan(2x)}\cdot \sec^2(2x)\cdot 2}{\frac{-1}{x^2}}$$

$$= \lim_{x\to 0^+} -\frac{\cos(2x)}{\sin(2x)}\cdot\frac{2x^2}{\cos^2(2x)}$$

$$= \lim_{x\to 0^+} \frac{-2x^2}{\sin(2x)\cos(2x)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0^+} \frac{-4x}{2\cos^2(2x) - 2\sin^2(2x)}$$

$$= 0$$

$$\therefore \lim_{x\to 0^+} (\tan(2x))^x = e^0 = \boxed{1}$$

(3) [5] Determine $f(x)$ if

$$f''(x) = 6x + \sin(x), \quad f(0) = 0, \quad f'(0) = 2$$

$$f'(x) = 3x^2 - \cos(x) + C_1$$

$$f'(0) = 2 \Rightarrow 2 = 3(\underbrace{\cancel{0}}_{0})^2 - \underbrace{\cancel{\cos}(0)}_{1} + C_1$$

$$\therefore C_1 = 2 + 1 = 3.$$

$$\therefore f'(x) = 3x^2 - \cos(x) + 3$$

$$f(x) = x^3 - \sin(x) + 3x + C_2$$

$$f(0) = 0 \Rightarrow 0 = 0^3 - \sin(0) + 3(0) + C_2$$

$$\therefore C_2 = 0$$

$$\boxed{\therefore f(x) = x^3 + 3x - \sin(x)}$$