Question 1 [10 points]:

(a) Set up BUT DO NOT EVALUATE the integral representing the area bounded between the curves $y = \sqrt{4 - x^2}$ (the top half of a circle) and $y = x^2 - 4$ over $-2 \leq x \leq 2$.

[3]

(b) Use T_4, the trapezoid rule on four subintervals, to approximate the integral in part (a).

[5]

(c) Is your approximation in part (b) an overestimate or underestimate of the exact area? Explain.

[2]

Question 2 [10 points]:

(a) Evaluate the improper integral or show that is it divergent. Clearly and neatly show all details, including any required substitutions or limits.

$$\int_0^1 \frac{e^x}{\sqrt{e^x - 1}}\, dx$$

[5]

(b) Use the comparison theorem to determine whether $\displaystyle\int_1^\infty \frac{x}{e^x + x^4}\, dx$ converges or diverges. Be sure to state any theorems or results used.

[5]

Question 3 [10 points]:

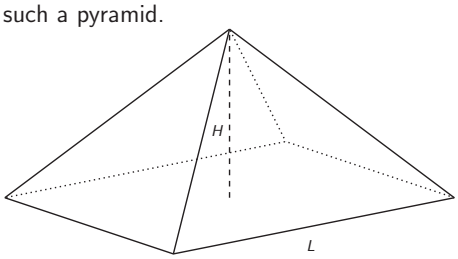
(a) The region in the first quadrant that is bounded by $y = x(2 - x)$, $y = 2 - x^2$ and the y-axis is rotated about the x-axis. Determine the volume of the resulting solid. (The washer method would be best here.)

[5]

(b) The same region as in part (a) is rotated about the vertical line $x = 1$. Determine the volume of the resulting solid. (Cylindrical shells would be best here.)

[5]

Question 4 [5 points]:
A pyramid has total height H and a square base of side length L. Use integration to determine the volume of such a pyramid.



[5]

Question 5 [5 points]: Determine the area of the region in the first quadrant that is bounded by $y = \arcsin(x)$, $y = \arccos(x)$ and the x axis. (Hint: think about x as a function of y.)

[5]

Question 6 [5 points]: **DO ONE OF (a) AND (b)**

(a) A cylindrical vessel of top radius 2 m and height 3 m is filled with water to a depth of 2 m. Determine the amount of work required to empty the vessel by pumping water up and over the top rim. Recall that the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2. You may leave the constants ρ and g in your final answer if you like. State units with your answer.

[5]

(b) A chain lying on the ground is 10 m long and has a mass of 80 kg. Use integration to determine the amount of work required to raise one end of the chain to a height of 6 m . Again, recall that acceleration due to gravity is $g = 9.8$ m/s^2 (you may leave the constant g in your final answer if you like.)

[5]