**Question 1:**

**(a)[3]** Determine $\int e^{3x}\cos(e^{3x})\,dx = I$

Let $u = e^{3x}$

$du = 3e^{3x}\,dx$

$$I = \frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(u) + C$$

$$= \boxed{\frac{1}{3}\sin(e^{3x}) + C}$$

**(b)[3]** Evaluate $\int_{-\sqrt{8}}^{\sqrt{8}} \frac{9x}{\sqrt{x^2+1}}\,dx = I$ $\Big\} \leftarrow$ notice integrand is odd on $[-\sqrt{8}, \sqrt{8}]$, so integral will be $\boxed{\text{zero}}$.

Let $u = x^2+1$, $du = 2x\,dx$ $\Big\}$ $x = -\sqrt{8} \Rightarrow u = 9$, $x = \sqrt{8} \Rightarrow u = 9$

$$\therefore \quad I = \frac{9}{2}\int_9^9 \frac{1}{\sqrt{u}}\,du = \boxed{0}$$

**(c)[4]** Suppose $\int_a^b f(x)\,dx = b$ and $\int_a^b g(x)\,dx = a$. What is the average value of $f(x) - g(x)$ over the interval $[a, b]$? Simplify your final answer.

$$(f-g)_{avg} = \frac{1}{b-a}\int_a^b (f(x) - g(x))\,dx$$

$$= \frac{1}{b-a}\left[\int_a^b f(x)\,dx - \int_a^b g(x)\,dx\right]$$

$$= \frac{1}{b-a}[b-a] = \boxed{1}$$

**Question 2:**

**(a)[4]** Determine $\int \arccos(t)\,dt = I$

Let $u = \arccos(t)$ $\quad dv = dt$

$du = \frac{-1}{\sqrt{1-t^2}}\,dt$ $\quad v = t$

$$I = \int u\,dv = uv - \int v\,du = t\arccos(t) - \int t\left(\frac{-1}{\sqrt{1-t^2}}\right)dt$$

$$= t\arccos(t) - \frac{1}{2}\int \frac{-2t}{\sqrt{1-t^2}}\,dt \quad u = 1-t^2, \; du = -2t\,dt$$

$$= t\arccos(t) - \sqrt{1-t^2} + C$$

**(b)[6]** Determine $\int x(\ln(x))^2\,dx = I$

Let $u = (\ln(x))^2$; $dv = x\,dx$

$du = 2\ln(x)\cdot\frac{1}{x}dx$; $v = \frac{x^2}{2}$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= (\ln(x))^2\frac{x^2}{2} - \int \frac{x^2}{\cancel{2}}\cdot\cancel{2}\ln(x)\cdot\frac{1}{x}\,dx$$

$$= (\ln(x))^2\frac{x^2}{2} - \underbrace{\int x\ln(x)\,dx}_{\substack{u=\ln(x) \quad dv = x\,dv \\ du = \frac{1}{x}dx \quad v = \frac{x^2}{2}}}$$

$$= (\ln(x))^2\frac{x^2}{2} - \left[\frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\,\frac{1}{x}\,dx\right]$$

$$= \boxed{(\ln(x))^2\frac{x^2}{2} - \frac{x^2}{2}\ln(x) + \frac{x^2}{4} + C}$$

**Question 3:**

**(a)[3]** Evaluate $\int_0^1 \sin^2(\pi x)\,dx = I$

$$I = \int_0^1 \frac{1}{2} - \frac{\cos(2\pi x)}{2}\,dx$$

$$= \left[\frac{x}{2}\right]_0^1 - \cancel{\left[\frac{\sin(2\pi x)}{4\pi}\right]_0^1} \rightarrow 0$$

$$= \boxed{\frac{1}{2}}$$

**(b)[7]** Determine $\int \tan^3(x)\sec^7(x)\,dx = I$

$$I = \int \tan^2(x)\sec^6(x)\;\sec(x)\tan(x)\,dx$$

$$= \int [\sec^2(x) - 1]\sec^6(x)\,\sec(x)\tan(x)\,dx \quad \left\{ \begin{array}{l} \text{let } u = \sec(x) \\ du = \sec(x)\tan(x)\,dx \end{array} \right.$$

$$= \int (u^2-1)\,u^6\,du$$

$$= \int u^8 - u^6\,du$$

$$= \frac{u^9}{9} - \frac{u^7}{7} + C$$

$$= \boxed{\frac{\sec^9(x)}{9} - \frac{\sec^7(x)}{7} + C.}$$

**Question 4 [10 points]:** Determine $\int \frac{\sqrt{16-x^2}}{x^2}\,dx = I$

Let $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$$I = \int \frac{\sqrt{16-16\sin^2\theta}}{16\sin^2\theta}\, 4\cos\theta\, d\theta$$

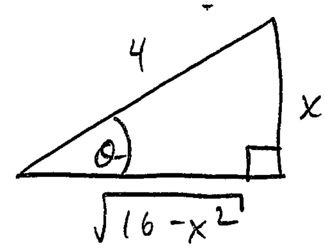
$$= \int \frac{4\cos\theta \cdot 4\cos\theta}{16\sin^2\theta}\, d\theta$$

$$= \int \cot^2\theta\, d\theta$$

$$= \int (\csc^2\theta - 1)\, d\theta$$

$$= -\cot\theta - \theta + C$$

$$= \boxed{-\frac{\sqrt{16-x^2}}{x} - \arcsin\left(\frac{x}{4}\right) + C}$$



**Question 5 [10 points]:** Determine $\int \frac{1}{(x+2)(x-1)^2}\,dx$ .

$$\frac{1}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x+2)(x-1) + C(x+2)}{(x+2)(x-1)^2}$$

$$= \frac{(A+B)x^2 + (-2A+B+C)x + (A-2B+2C)}{(x+2)(x-1)^2}$$

$\therefore$ ① $A+B=0$
② $-2A+B+C=0$
③ $A-2B+2C=1$

① $\Rightarrow$ $B=-A$
② $\Rightarrow$ $C = 2A-B = 3A$
③ $\Rightarrow$ $A-2(-A)+2(3A)=1$
$9A=1$
$A=\frac{1}{9}$

$\therefore$ $B = \frac{-1}{9}$
$C = 3A = \frac{1}{3}$

$$\therefore \quad I = \int \frac{(1/9)}{x+2} + \frac{(-1/9)}{x-1} + \frac{(1/3)}{(x-1)^2}\,dx$$

$$= \boxed{\frac{1}{9}\ln|x+2| - \frac{1}{9}\ln|x-1| + \frac{1}{3}\,\frac{1}{x-1} + C}$$