Question 1:

(a)[4] Suppose $\int_a^b f(x)\,dx = a$ and $\int_a^b g(x)\,dx = b$. What is the average value of $f(x) - g(x)$ over the interval $[a, b]$? Simplify your final answer.

$$(f-g)_{avg} = \frac{1}{b-a}\int_a^b f(x) - g(x)\,dx$$

$$= \frac{1}{b-a}\left[\int_a^b f(x)\,dx - \int_a^b g(x)\,dx\right]$$

$$= \frac{1}{b-a}\left[a - b\right] = \boxed{-1}$$

(b)[3] Determine $\int e^{2x}\sin(e^{2x})\,dx$.

Let $u = e^{2x}$
$du = 2e^{2x}\,dx$

$$\int e^{2x}\sin(e^{2x})\,dx = \frac{1}{2}\int \sin(u)\,du$$

$$= \frac{-1}{2}\cos(u) + C$$

$$= \boxed{\frac{-1}{2}\cos(e^{2x}) + C}$$

(c)[3] Evaluate $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx$. ← notice integrand is odd on $[-\sqrt{3}, \sqrt{3}]$ so integral will be $\boxed{\text{zero.}}$

Let $u = x^2+1$, $du = 2x\,dx$ $\Big\}$ $x = -\sqrt{3} \Rightarrow u = 4$, $x = \sqrt{3} \Rightarrow u = 4$

$$\therefore \int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx = 2\int_4^4 \frac{1}{\sqrt{u}}\,du = \boxed{0}$$

Question 2:

(a)[4] Determine $\int \arctan(t)\,dt = I$

Let $u = \arctan(t)$ $\quad dv = dt$

$du = \frac{1}{1+t^2}\,dt \quad v = t$

$$I = \int u\,dv = t\arctan(t) - \int \frac{t}{1+t^2}\,dt$$

now let $u = 1+t^2$

$du = 2t\,dt$

$$= \boxed{t\arctan(t) - \frac{1}{2}\ln|1+t^2| + C}$$

(b)[6] Determine $\int x(\ln(x))^2\,dx = I$

Let $u = (\ln(x))^2 \quad dv = x\,dx$

$du = 2\ln(x)\frac{1}{x}\,dx \quad v = \frac{x^2}{2}$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= (\ln(x))^2\frac{x^2}{2} - \int \frac{x^2}{\cancel{2}}\cancel{2}\ln(x)\frac{1}{x}\,dx$$

$$= (\ln(x))^2\frac{x^2}{2} - \underbrace{\int x\ln(x)\,dx}$$

$u = \ln(x);\ dv = x\,dx$

$du = \frac{1}{x}\,dx;\ v = \frac{x^2}{2}$

$$= (\ln(x))^2\frac{x^2}{2} - \left[\frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\frac{1}{x}\,dx\right]$$

$$= \boxed{(\ln(x))^2\frac{x^2}{2} - \frac{x^2}{2}\ln(x) + \frac{x^2}{4} + C}$$

Question 3:

(a)[7] Determine $\int \tan^3(x)\sec^5(x)\,dx$.

$$I = \int \tan^2(x)\sec^4(x)\ \sec(x)\tan(x)\,dx$$

$$= \int [\sec^2(x)-1]\sec^4(x)\sec(x)\tan(x)\,dx \quad \left.\begin{array}{l} \text{let } u=\sec(x) \\ du = \sec(x)\tan(x)dx \end{array}\right.$$

$$= \int (u^2-1)u^4\,du$$

$$= \int u^6 - u^4\,du$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \boxed{\frac{\sec^7(x)}{7} - \frac{\sec^5(x)}{5} + C}$$

(b)[3] Evaluate $\int_0^1 \cos^2(\pi x)\,dx = I$

$$I = \int_0^1 \frac{1}{2} + \frac{\cos(2\pi x)}{2}\,dx$$

$$= \left[\frac{x}{2}\right]_0^1 + \cancel{\left[\frac{\sin(2\pi x)}{4\pi}\right]_0^1} \rightarrow 0$$

$$= \boxed{\frac{1}{2}}$$

Question 4 [10 points]: Determine $\int \frac{\sqrt{9-x^2}}{x^2}\,dx = I$

Let $x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$I = \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta}\,3\cos\theta\,d\theta$$

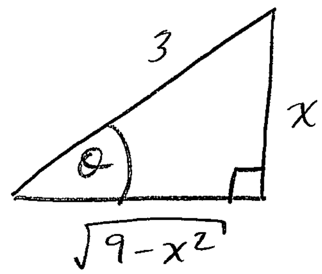
$$= \int \frac{\cancel{3}\cos\theta \cdot \cancel{3}\cos\theta}{\cancel{9}\sin^2\theta}\,d\theta$$

$$= \int \cot^2\theta\,d\theta$$

$$= \int \csc^2\theta - 1\,d\theta$$

$$= -\cot\theta - \theta + C$$

$$= \boxed{-\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C}$$



Question 5 [10 points]: Determine $\int \frac{1}{(x-2)(x+1)^2}\,dx$.

$$\frac{1}{(x-2)(x+1)^2} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + B(x-2)(x+1) + C(x-2)}{(x-2)(x+1)^2}$$

$$= \frac{(A+B)x^2 + (2A-B+C)x + (A-2B-2C)}{(x-2)(x+1)^2}$$

$$\left.\begin{array}{ll} ① & A+B=0 \\ ② & 2A-B+C=0 \\ ③ & A-2B-2C=1 \end{array}\right\} \begin{array}{l} ① \Rightarrow B=-A \\ ② \Rightarrow C = B-2A = -3A \\ ③ \Rightarrow A-2(-A)-2(-3A)=1 \\ \qquad 9A = 1 \\ \qquad A = \frac{1}{9} \end{array} \left.\right\} \begin{array}{l} \therefore\ B = -A = -\frac{1}{9} \\ C = -3A = -\frac{1}{3} \end{array}$$

$$\therefore\ I = \int \frac{(1/9)}{x-2} + \frac{(-1/9)}{x+1} + \frac{(-1/3)}{(x+1)^2}\,dx$$

$$= \boxed{\frac{1}{9}\ln|x-2| - \frac{1}{9}\ln|x+1| + \frac{1}{3}\,\frac{1}{x+1} + C}$$