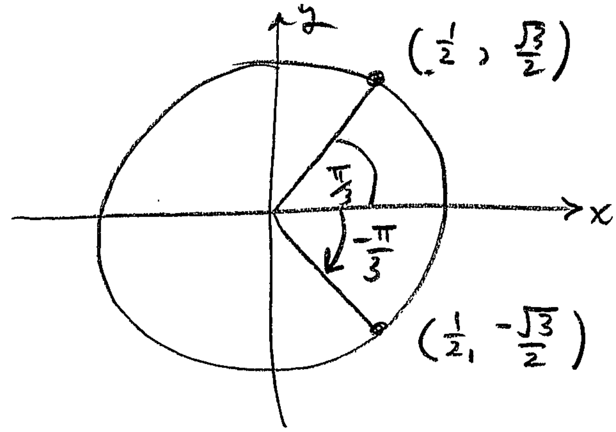
Question 1:

(a)[3] Determine $\sin^{-1}(-\sqrt{3}/2)$.

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = angle θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin\theta = -\frac{\sqrt{3}}{2}$

$= \boxed{-\frac{\pi}{3}}$

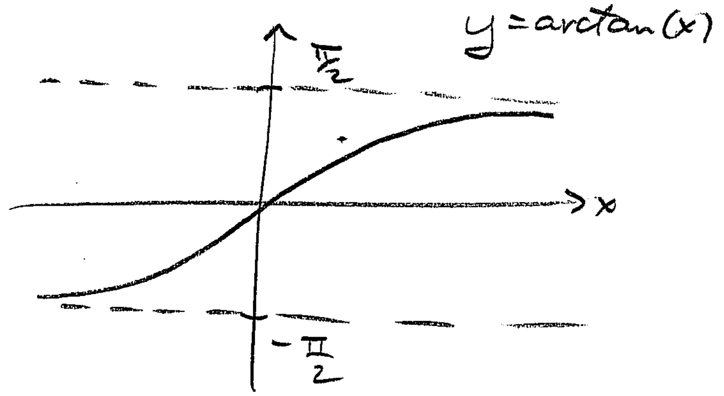


(b)[3] Determine $\lim_{x \to -\infty} \arctan(x^3)$.

As $x \to -\infty$, $x^3 \to -\infty$,

so $\arctan(x^3) \to -\frac{\pi}{2}$

$\therefore \lim_{x \to -\infty} \arctan(x^3) = \boxed{-\frac{\pi}{2}}$



(c)[4] Let $f(x) = x \arccos(x) - \sqrt{1-x^2}$. Determine $f'(1/2)$.

$f'(x) = \arccos(x) - \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{1}{2\sqrt{1-x^2}}(-2x)}$

$f'\left(\frac{1}{2}\right) = \arccos\left(\frac{1}{2}\right)$

$= \boxed{\frac{\pi}{3}}$

Question 2:

(a)[3] Evaluate the limit if it exists: $\lim_{x\to 0} \frac{\sinh(5x)}{\tanh(3x)}$. $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0} \frac{5\cosh(5x)}{3\operatorname{sech}^2(3x)}$$

$$= \boxed{\frac{5}{3}}$$

(b)[3] Determine $f'(0)$ if $f(x) = \ln(\cosh(2x)) - \operatorname{sech}(\ln(1+x))$.

$$f'(x) = \frac{2\sinh(2x)}{\cosh(2x)} + \operatorname{sech}(\ln(1+x))\tanh(\ln(1+x)) \cdot \frac{1}{1+x}$$

$$f'(0) = \cancel{\frac{2\sinh(0)}{\cosh(0)}}^{\to 0} + \operatorname{sech}(\ln(1))\cancel{\tanh(\ln(1))}^{\to 0} \cdot \frac{1}{1+0}$$

$$= \boxed{0}$$

(c)[4] Does the equation $\sinh(x) = 1 - \cosh(x)$ have solutions? If so, find them. If not, explain why.

$$\sinh(x) = 1 - \cosh(x)$$

$$\Rightarrow \frac{e^x - e^{-x}}{2} = 1 - \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow e^x - \cancel{e^{-x}} = 2 - e^x \cancel{- e^{-x}}$$

$$\Rightarrow 2e^x = 2$$

$$e^x = 1$$

$$\boxed{x = 0}$$

Question 3:

(a)[5] Evaluate the limit if it exists: $\lim\limits_{x\to 0} \frac{x^2}{1+x/2-\sqrt{1+x}}$ $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0} \frac{2x}{\frac{1}{2} - \frac{1}{2}(1+x)^{-\frac{1}{2}}} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{2}{\frac{1}{4}(1+x)^{-3/2}}$$

$$= \boxed{8}$$

(b)[5] Evaluate the limit if it exists: $\lim\limits_{x\to 0^+} x(\ln(x))^2$ $\sim$ "$0 \cdot \infty$"

$$= \lim_{x\to 0^+} \frac{(\ln(x))^2}{\frac{1}{x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{2\ln(x)\cdot\frac{1}{x}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2\ln(x)}{\frac{1}{x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-2\left(\frac{1}{x}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} 2x = \boxed{0}$$

Question 4:

(a)[5] Evaluate the limit if it exists: $\lim_{x\to 0} \dfrac{x - \sin(x)}{\sin(x^2)}$ $\sim$ "$\dfrac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0} \frac{1-\cos x}{2x\cos(x^2)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{\sin x}{2\cos(x^2) - 4x^2\sin(x^2)}$$

$$= \boxed{0}$$

(b)[5] Evaluate the limit if it exists: $\lim_{x\to\infty} x^{(e^{-x})}$ $\sim$ "∞^0"

$$\lim_{x\to\infty} x^{(e^{-x})} = \lim_{x\to\infty} e^{\underbrace{(e^{-x})\ln x}_{\text{examine this!}}}$$

$$\lim_{x\to\infty} e^{-x}\cdot\ln x = \lim_{x\to\infty} \frac{\ln x}{e^x} \sim \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{1/x}{e^x} = 0$$

$$\therefore \lim_{x\to\infty} x^{(e^{-x})} = e^0 = \boxed{1}$$

Question 5: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_{-2}^{1} 2x^2\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplication.

$[a,b] = [-2,1]$, $f(x) = 2x^2$.

$$\Delta x = \frac{b-a}{n} = \frac{1-(-2)}{n} = \frac{3}{n}.$$

$$x_i = a + i\Delta x = -2 + i\left(\frac{3}{n}\right)$$

$$f(x_i) = 2\left[-2 + i\left(\frac{3}{n}\right)\right]^2 = 2\left[4 - 12\frac{i}{n} + 9\frac{i^2}{n^2}\right]$$

$$\int_{-1}^{1} 2x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 2\left[4 - 12\frac{i}{n} + 9\frac{i^2}{n^2}\right]\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{6}{n}\right)\left[\left(\sum_{i=1}^{n} 4\right) - \left(\frac{12}{n}\right)\left(\sum_{i=1}^{n} i\right) + \left(\frac{9}{n^2}\right)\left(\sum_{i=1}^{n} i^2\right)\right]$$

$$= \lim_{n\to\infty} \left(\frac{6}{n}\right)\left[4n - \frac{12}{n}\cdot\frac{n(n+1)}{2} + \frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[24 - 36\cdot\underbrace{\frac{n+1}{n}}_{\to 1} + 9\cdot\underbrace{\frac{n+1}{n}}_{\to 1}\cdot\underbrace{\frac{2n+1}{n}}_{\to 2}\right]$$

$$= \boxed{6}$$