Question 1:

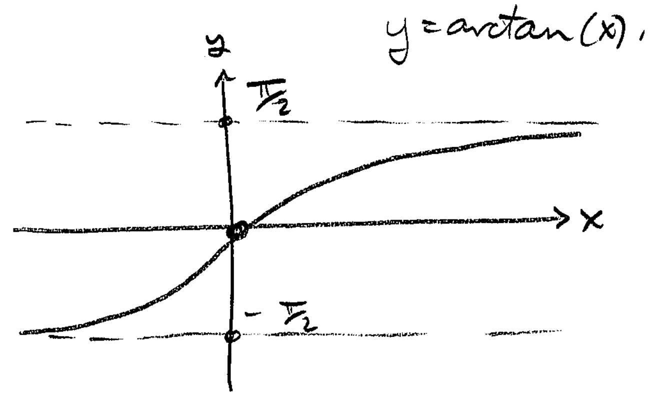
(a)[3] Determine $\lim_{x\to-\infty} \arctan(x^2)$.

As $x \to -\infty$,

$x^2 \to \infty$,

so $\arctan(x^2) \to \frac{\pi}{2}$

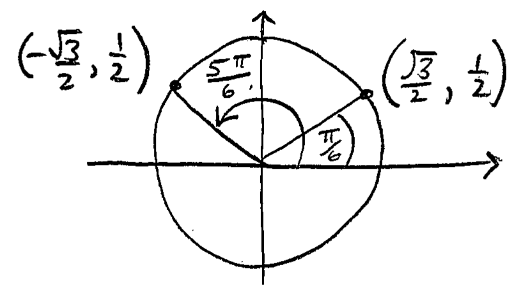
$\therefore \lim_{x\to-\infty} \arctan(x^2) = \boxed{\frac{\pi}{2}}$



(b)[3] Determine $\cos^{-1}(-\sqrt{3}/2)$.

$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ = angle θ in $[0,\pi]$ such that $\cos\theta = -\frac{\sqrt{3}}{2}$

$= \boxed{\frac{5\pi}{6}}$,



(c)[4] Let $f(x) = x\arcsin(x) + \sqrt{1-x^2}$. Determine $f'(1/2)$.

$f'(x) = \arcsin(x) + \cancel{\frac{x}{\sqrt{1-x^2}}} + \frac{1}{\cancel{2}} \cancel{\frac{1}{\sqrt{1-x^2}} \cdot (-\cancel{2}x)}$

$f'(\tfrac{1}{2}) = \arcsin(\tfrac{1}{2}) = \boxed{\frac{\pi}{6}}$

Question 2:

(a)[3] Evaluate the limit if it exists: $\lim_{x\to 0} \frac{\tanh(3x)}{\sinh(5x)}$. $\sim$ "$\frac{0}{0}$"

$$\stackrel{H}{=} \lim_{x\to 0} \frac{\operatorname{sech}^2(3x)\cdot 3}{\cosh(5x)\cdot 5}$$

$$= \boxed{\frac{3}{5}}$$

(b)[3] Determine $f'(0)$ if $f(x) = \ln(\cosh(2x)) - \operatorname{sech}(\ln(1+x))$.

$$f'(x) = \frac{2\sinh(2x)}{\cosh(2x)} + \operatorname{sech}(\ln(1+x))\cdot\tanh(\ln(1+x))\cdot\frac{1}{1+x}$$

$$f'(0) = \cancel{\frac{2\sinh(0)}{\cosh(0)}}^{\to 0} + \cancel{\operatorname{sech}(\ln(1))}^{\to 1}\cdot\cancel{\tanh(\ln(1))}^{\to 0}\cdot\frac{1}{1+0}$$

$$= \boxed{0}$$

(c)[4] Does the equation $\sinh(x) = \cosh(x)$ have solutions? If so, find them. If not, explain why.

No solution: $\sinh(x) = \cosh(x)$

$$\Leftrightarrow \frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x}}{2}$$

$$\Leftrightarrow 2e^{-x} = 0$$

$$\Leftrightarrow e^{-x} = 0$$ which is not satisfied by any x.

Question 3:

(a)[5] Evaluate the limit if it exists: $\lim_{x\to 0} \frac{x-\sin(x)}{\sin(x^2)}$ $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0} \frac{1-\cos(x)}{\cos(x^2)\cdot 2x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{+\sin(x)}{-\sin(x^2)\cdot 4x^2 + \cos(x^2)\cdot 2}$$

$$= \boxed{0}$$

(b)[5] Evaluate the limit if it exists: $\lim_{x\to 0} \frac{\sqrt{1+x}-1-x/2}{x^2}$ $\sim$ "$\frac{0}{0}$"

$$\overset{H}{=} \lim_{x\to 0} \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2}}{2x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{-\frac{1}{4}(1+x)^{-\frac{3}{2}}}{2}$$

$$= \boxed{-\frac{1}{8}}$$

Question 4:

(a)[5] Evaluate the limit if it exists: $\lim_{x\to 0^+} x(\ln(x))^2$ $\sim$ "$0 \cdot \infty$"

$$= \lim_{x\to 0^+} \frac{(\ln(x))^2}{x^{-1}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{2\ln(x)\cdot\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2\ln(x)}{\frac{1}{x}} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 2x = \boxed{0}$$

(b)[5] Evaluate the limit if it exists: $\lim_{x\to\infty} x^{(e^{-x})}$ $\sim$ "∞^0"

$$\lim_{x\to\infty} x^{(e^{-x})} = \lim_{x\to\infty} e^{\underbrace{[(e^{-x})\ln x]}_{\text{examine this.}}}$$

$$\lim_{x\to\infty} e^{-x}\ln x = \lim_{x\to\infty} \frac{\ln x}{e^x} \sim \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{e^x}$$

$$= \lim_{x\to\infty} \frac{1}{xe^x} = 0$$

$$\therefore \lim_{x\to\infty} x^{(e^{-x})} = e^0 = \boxed{1}.$$

Question 5: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_{-1}^{2} 3x^2\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplication.

$[a,b] = [-1,2]$, $f(x) = 3x^2$.

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$.

$x_i = a + i\Delta x = -1 + i\left(\frac{3}{n}\right)$

$f(x_i) = 3\left[-1 + i\left(\frac{3}{n}\right)\right]^2 = 3\left[1 - 6\frac{i}{n} + \frac{9i^2}{n^2}\right]$

$$\int_{-1}^{2} 3x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} 3\left[1 - 6\frac{i}{n} + \frac{9i^2}{n^2}\right]\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{9}{n}\right)\left[\left(\sum_{i=1}^{n} 1\right) - \left(\frac{6}{n}\right)\left(\sum_{i=1}^{n} i\right) + \left(\frac{9}{n^2}\right)\left(\sum_{i=1}^{n} i^2\right)\right]$$

$$= \lim_{n\to\infty}\left(\frac{9}{n}\right)\left[n - \left(\frac{6}{n}\right)\left(\frac{n(n+1)}{2}\right) + \left(\frac{9}{n^2}\right)\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$$

$$= \lim_{n\to\infty}\left[9 - 27\cdot\underbrace{\frac{n+1}{n}}_{\to 1} + \frac{27}{2}\cdot\underbrace{\frac{n+1}{n}}_{\to 1}\cdot\underbrace{\frac{2n+1}{n}}_{\to 2}\right]$$

$$= \boxed{9}$$