(1) [7 points] Use Simpson's rule with $n = 4$ to approximate $\int_0^{2\pi} x\cos(x)\,dx$. Simplify your final answer.

$$\Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$$

$$S_4 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{\pi}{6}\left[0\underbrace{\cos(0)}_{0} + 4\left(\frac{\pi}{2}\right)\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + 2\pi\underbrace{\cos(\pi)}_{-1} + 4\left(\frac{3\pi}{2}\right)\underbrace{\cos\left(\frac{3\pi}{2}\right)}_{0} + 2\pi\underbrace{\cos(2\pi)}_{1}\right]$$

$$= \frac{\pi}{6}\left[-2\pi + 2\pi\right]$$

$$= \boxed{0}$$

 Name: SOLUTIONS.

(2) [8 points] Evaluate the improper integral $\int_{-\infty}^{0} x^2 e^{x^3}\, dx$. Clearly and neatly show all details, including any required substitutions or limits.

$$\int_{-\infty}^{0} x^2 e^{x^3} dx = \lim_{a \to -\infty} \int_a^0 x^2 e^{x^3} dx$$

$u = x^3$

$du = 3x^2 dx$

$$= \lim_{a \to -\infty} \left[\frac{e^{x^3}}{3} \right]_a^0$$

$$= \lim_{a \to -\infty} \left[\frac{e^0}{3} - \underbrace{\frac{e^{a^3}}{3}}_{\to 0} \right]$$

$$= \boxed{\frac{1}{3}}$$