(1) [7 points] Use Simpson's rule with $n = 4$ to approximate $\int_0^{2\pi} x \sin(x)\, dx$. Simplify your final answer.

$$\Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$$

$$S_4 = \frac{\Delta x}{3}\left[f(x_0) + 4 f(x_1) + 2 f(x_2) + 4 f(x_3) + f(x_4)\right]$$

$$= \frac{\pi}{6}\left[0 \overset{0}{\cancel{\sin(0)}} + 4\left(\frac{\pi}{2}\right) \overset{1}{\cancel{\sin\left(\frac{\pi}{2}\right)}} + \cancel{2\pi \sin}\overset{0}{(\pi)} + 4\left(\frac{3\pi}{2}\right) \overset{-1}{\cancel{\sin\left(\frac{3\pi}{2}\right)}} + 2\pi \underset{0}{\cancel{\sin(2\pi)}}\right]$$

$$= \frac{\pi}{6}\left[2\pi - 6\pi\right]$$

$$= \boxed{-\frac{2\pi^2}{3}}$$

(2) [8 points] Evaluate the improper integral $\int_{-\infty}^{0} xe^{-x^2}\,dx$. Clearly and neatly show all details, including any required substitutions or limits.

$$\int_{-\infty}^{0} x e^{-x^2} dx = \lim_{a\to-\infty} \int_a^0 x e^{-x^2} dx$$

$u = -x^2$
$du = -2x\,dx$

$$= \lim_{a\to-\infty} \left[\frac{e^{-x^2}}{-2} \right]_a^0$$

$$= \lim_{a\to-\infty} \left[\frac{e^0}{-2} - \underbrace{\left(\frac{e^{-a^2}}{-2} \right)}_{\to 0} \right]$$

$$= \boxed{-\frac{1}{2}}$$