(1) [7] Evaluate $\int_0^{\pi/4} \sec^4(x)\tan^6(x)\,dx$

$$I = \int_0^{\pi/4} \sec^2(x)\tan^6(x)\sec^2(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} \left(1+\tan^2(x)\right)\tan^6(x)\sec^2(x)\,dx$$

let $u = \tan(x)$, $du = \sec^2(x)\,dx$ $\Big\}$ $x=0 \Rightarrow u=0$, $x=\frac{\pi}{4} \Rightarrow u=\tan\left(\frac{\pi}{4}\right)=1$

$$\therefore I = \int_0^1 (1+u^2)u^6\,du$$

$$= \int_0^1 (u^6+u^8)\,du$$

$$= \left[\frac{u^7}{7}+\frac{u^9}{9}\right]_0^1$$

$$= \frac{1}{7}+\frac{1}{9}$$

$$= \boxed{\frac{16}{63}}$$

(2) [8] Determine $\displaystyle\int \frac{1}{x\sqrt{16-x^2}}\,dx$

$$I = \int \frac{1}{x\sqrt{16-x^2}}\,dx \quad : \text{let } x = 4\sin\theta$$

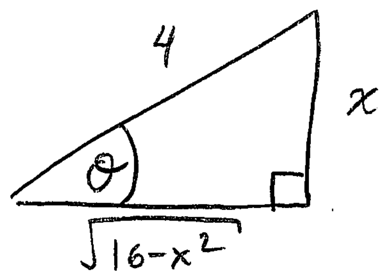
$$dx = 4\cos\theta\,d\theta$$

$$= \int \frac{1}{\cancel{4}\sin\theta\sqrt{16-16\sin^2\theta}} \cdot \cancel{4}\cos\theta\,d\theta$$

$$= \frac{1}{4}\int \frac{1}{\sin\theta\,\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\,d\theta$$

$$= \frac{1}{4}\int \csc\theta\,d\theta$$

$$= \frac{1}{4}\ln\left|\csc\theta - \cot\theta\right| + C$$



$$= \boxed{\frac{1}{4}\ln\left|\frac{4}{x} - \frac{\sqrt{16-x^2}}{x}\right| + C}$$