Question 1:

(a)[5] Solve the following system of equations:

$$x - 2y + z = 3$$
$$3x - 5y + 6z = 14$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 3 & -5 & 6 & 14 \end{array}\right]$$

$R_2 = (-3)r_1 + r_2$:

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 3 & 5 \end{array}\right]$$

$R_1 = 2r_2 + r_1$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 13 \\ 0 & 1 & 3 & 5 \end{array}\right]$$

$\therefore$ $y = 5 - 3z$

$x = 13 - 7z$

z is any real number

(b)[2] State the solution for the system of equations which has the following REF matrix (the variables here are x, y, z and w):

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 7 \end{array}\right]$$

no solutions.

(c)[3] Perform one row operation to put the following matrix in REF and state the solution of the corresponding system (the variables here are x and y):

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \\ -1/2 & 1 & -5/2 \end{array}\right]$$

$R_3 = \frac{1}{2}r_1 + r_3$: $\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$ $y = -2$

$x = 5 + 2y = 5 + 2(-2) = 1$

$\boxed{\therefore\ x = 1,\ y = -2}$

Question 2: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -1 & 1 & 1 \\ 4 & 0 & 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} -3 & 2 \end{bmatrix}$$

Perform the following matrix calculations, if possible. If a calculation is not defined then state "not defined":

(a)[2] $\mathbf{CD} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \end{bmatrix} = \begin{bmatrix} -15 & 10 \\ -9 & 6 \end{bmatrix}$.

(b)[2] $\mathbf{DC} = \begin{bmatrix} -3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \end{bmatrix}$.

(c)[3] $\mathbf{B}(2\mathbf{A} - 3\mathbf{I}_3) = \begin{bmatrix} -1 & 1 & 1 \\ 4 & 0 & 2 \end{bmatrix} \left(2 \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 4 & 6 \\ -6 & 1 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -5 & -5 \\ -4 & 12 & 22 \end{bmatrix}$$

(d)[2] $(\underbrace{2\mathbf{A}}_{3\times 3} - \underbrace{3\mathbf{B}}_{2\times 3})$ not defined.

(e)[1] Suppose there is a matrix **P** such that the product **BAPC** is defined. What must be the dimension (or size) of **P**?

$$\underset{2\times 3}{B}\ \underset{3\times 3}{A}\ \underset{3\times 2}{P}\ \underset{2\times 1}{C}$$

P must be 3×2.

Question 3:

(a)[7] Determine $\mathbf{A}^{-1}$ where $\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = r_1 + r_2$:

$R_3 = (-1)r_1 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 = r_2 + r_1$:

$R_3 = (-1)r_2 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 1 \end{array}\right]$$

$R_3 = (-1)r_3$: $\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$

$R_1 = (-3)r_3 + r_1$:

$R_2 = (-3)r_3 + r_2$: $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -2 & 3 \\ 0 & 1 & 0 & -5 & -2 & 3 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$

$\therefore A^{-1} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}$

(b)[3] Show that the matrix $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}$ does not have an inverse.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -3 & -6 & 0 & 1 \end{array}\right]$$

$R_2 = 3r_1 + r_2$:

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

does not reduce to I_2, so B does not have an inverse.

Question 4 [10 points]: A company makes two kinds of animal food, type A and type B, which contain two food supplements. It takes 2 pounds of the first supplement and one pound of the second to make a dozen cans of food A. It takes 4 pounds of the first supplement and 5 pounds of the second to make a dozen cans of food B. On a certain day 80 pounds of the first supplement and 70 pounds of the second are available. How many dozen cans of each type of food should be made to maximize profit if profit on a dozen cans of type A is \$3.00 while profit on a dozen cans of type B is \$10.00?

Graph paper is provided on the next page. Carefully set up the problem, identify your variables, neatly sketch any required graphs and state a clear conclusion.

Let x = # dozen of type A
y = # dozen of type B.

Maximize $z = 3x + 10y$ } profit

Subject to
$2x + 4y \leq 80$ } first type of supplement
$x + 5y \leq 70$ } second type of supplement.
$x \geq 0$
$y \geq 0$

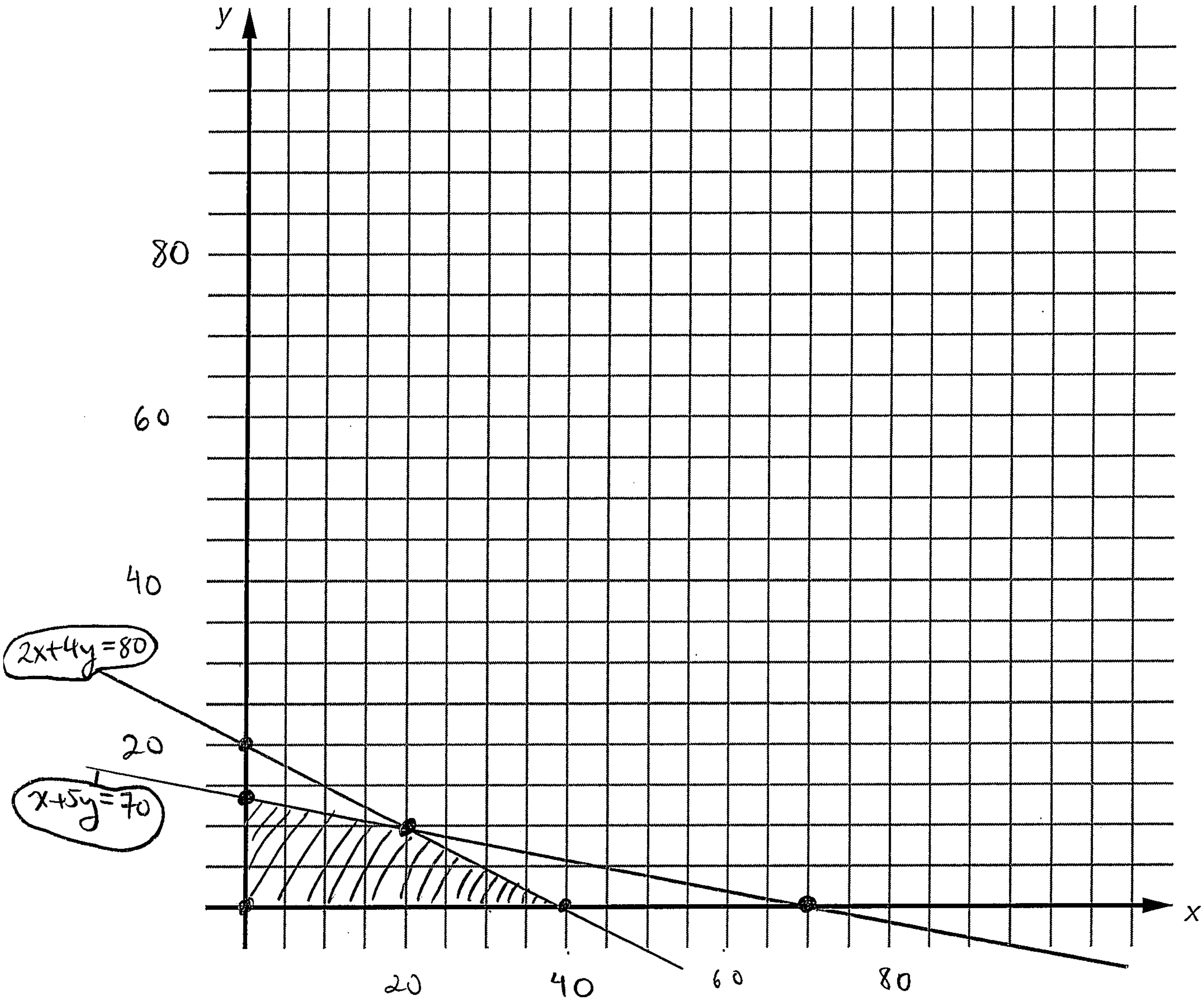
Inequality	Line	test pt.	test
$2x+4y \leq 80$	$2x+4y=80$	$(0,0)$	$0+0 \leq 80$: T
$x+5y \leq 70$	$x+5y=70$	$(0,0)$	$0+0 \leq 70$: T.

Corner points:
- By inspection: $(0,0)$, $(0,14)$, $(40,0)$
- Solving: $\left.\begin{array}{l} ① \ 2x+4y=80 \\ ② \ x+5y=70 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} ① \ x+2y=40 \\ ② \ x+5y=70 \end{array}\right\}$

$② - ①$: $\left.\begin{array}{l} 3y=30 \\ y=10 \end{array}\right\} \Rightarrow x = 40-2(10) = 20$

$\therefore (20,10)$

Question 4 (continued)



C.P.	$z = 3x + 10y$
(0,0)	z = 0
(0,14)	z = 140
(40,0)	z = 120
(20,10)	z = 160

∴ 20 dozen of type A and 10 dozen of type B should be made

Question 5:

(a)[3] An investor opens a new account and deposits \$700. After 11 months the account holds \$765. If no other deposits were made, what is the rate of simple interest? (Express your answer as a percentage rounded to 2 decimal places.)

$$A = P(1 + rt)$$

$$\therefore \; r = \left(\frac{A}{P} - 1\right)\left(\frac{1}{t}\right)$$

$$r = \left(\frac{765}{700} - 1\right)\left(\frac{1}{\left(\frac{11}{12}\right)}\right)$$

$$\boxed{r \doteq 10.13\%}$$

(b)[4] \$500 invested at 8% compounded quarterly has the same value after 3 years as \$$A$ invested at 7% compounded monthly. Determine A. (Round your answer to 2 decimal places.)

$$500\left(1 + \frac{0.08}{4}\right)^{(3)(4)} = A\left(1 + \frac{0.07}{12}\right)^{(3)(12)}$$

$$\therefore \; A = \frac{500\left(1 + \frac{0.08}{4}\right)^{12}}{\left(1 + \frac{0.07}{12}\right)^{36}} \doteq \boxed{\$514.32}$$

(c)[3] At what rate of interest compounded semiannually will an investment double in 8 years?

Solve $\cancel{P}\left(1 + \frac{r}{2}\right)^{(2)(8)} = 2\cancel{P}$

$$1 + \frac{r}{2} = 2^{\frac{1}{16}}$$

$$r = 2\left(2^{1/16} - 1\right)$$

$$\boxed{r \doteq 8.85\%}$$