Question 5 [10]: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to state the row operations used at each step and clearly state the final solution.

$$x + y + 13z = 6$$
$$x - 2y + 4z = 6$$
$$-2x + 6y - z = -10$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 13 & 6 \\ 1 & -2 & 4 & 6 \\ -2 & 6 & -1 & -10 \end{array}\right]$$

$R_2 = (-1)r_1 + r_2$:
$R_3 = 2r_1 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & 1 & 13 & 6 \\ 0 & -3 & -9 & 0 \\ 0 & 8 & +25 & 2 \end{array}\right]$$

$R_2 = \frac{-1}{3} r_2$:
$$\left[\begin{array}{ccc|c} 1 & 1 & 13 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 8 & 25 & 2 \end{array}\right]$$

$R_1 = (-1)r_2 + r_1$:
$R_3 = (-8)r_2 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 10 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 = (-10)r_3 + r_1$:
$R_2 = (-3)r_3 + r_2$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\boxed{\therefore \quad x = -14, \; y = -6, \; z = 2}$$

Question 1:

(a)[2] Determine the slope of the line through $(-3, 5)$ and $(2, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-5}{2-(-3)} = \frac{0}{5} = \boxed{0}$$

(b)[2] Determine the slope of the line through $(-5, -2)$ and $(-4, 11)$.

$$m = \frac{11-(-2)}{-4-(-5)} = \frac{13}{1} = \boxed{13}$$

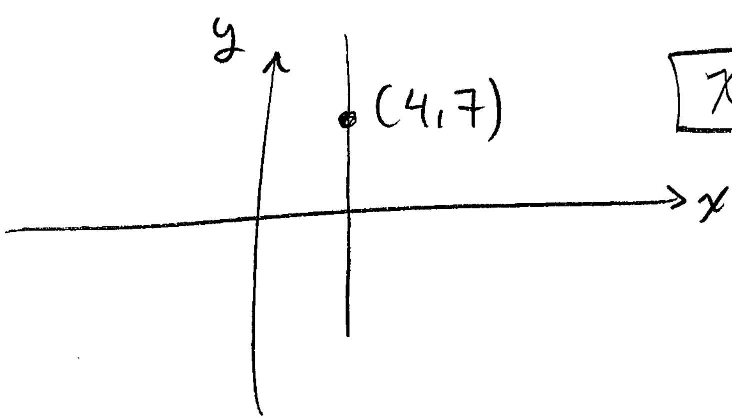
(c)[3] State the slope and y-intercept of the line $4x + 3y = 24$.

$$3y = -4x + 24$$

$$y = \frac{-4}{3}x + 8$$

$$\therefore m = \frac{-4}{3}, \quad y\text{-intercept } (0,8)$$

(d)[3] Determine an equation of the vertical line through $(4, 7)$.



$$\boxed{x=4}$$

Question 2:

(a)[5] Eight hundred people attend a basketball game, and total ticket sales are \$3102 . If adult tickets are \$6 and student tickets are \$3, determine the number of each type of ticket sold.

Let x = # adults
y = # students.

① $x+y=800$
② $6x+3y=3102$

① $\Rightarrow$ $y=800-x$
② $\Rightarrow$ $6x+3(800-x)=3102$

$$3x = 3102-2400$$
$$x=\frac{3102-2400}{3}=234$$
$$\therefore y = 800-234 = 566.$$

∴ 234 adult tickets and
566 student tickets were sold.

(b)[5] Determine an equation of the line through $(-5,2)$ and parallel to the line through $(1,2)$ and $(4,3)$.

Line through $(1,2)$ & $(4,3)$ has slope

$$m=\frac{3-2}{4-1}=\frac{1}{3}.$$

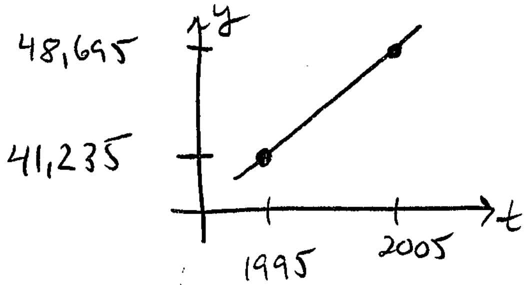
∴ Line through $(-5,2)$ also has slope $m=\frac{1}{3}$ (since parallel),
so line has equation $y-y_0=m(x-x_0)$

$$y-2=\frac{1}{3}(x-(-5))$$
$$\boxed{y-2=\frac{1}{3}(x+5)}$$

or $\boxed{y=\frac{1}{3}x+\frac{11}{3}}$

Question 3:

(a)[5] In 1995 there were 41,235 shopping centres in the United States. By 2005 there were 48,695. Find a linear equation relating the year x to the number of shopping centres y, and use your equation to predict the year in which the number of shopping centres will reach 60,000.



$$m = \frac{48{,}695 - 41{,}235}{2005 - 1995} = 746$$

$\therefore$ Equation of line is

$$\boxed{y - 41235 = 746\,(t - 1995)}$$

If $y = 60\,000$, $\quad 60\,000 - 41235 = 746(t - 1995)$

$$\Rightarrow t = \frac{60000 - 41235}{746} + 1995$$

$$\boxed{t \doteq 2020}$$

(b)[5] How many pounds of tea worth \$4.60 a pound should be mixed with tea worth \$6.50 a pound to get 10 pounds of blended tea worth \$5.74 a pound?

Let x = weight of tea worth 4.60 \$/lb.
y = weight of tea worth 6.50 \$/lb.

① $x + y = 10$
② $4.6x + 6.5y = (10)(5.74)$

① $\Rightarrow y = 10 - x$

② $\Rightarrow 4.6x + 6.5(10 - x) = 57.4$

$4.6x + 65 - 6.5x = 57.4$

$1.9x = 7.6$

$\Rightarrow x = 4$

$\therefore y = 10 - x = 10 - 4 = 6.$

$\therefore$ 4 pounds of the first tea should be mixed with 6 pounds of the second.

Question 4:

(a)[5] A company manufactures a certain product and sells it for \$550 per unit. The fixed cost is \$213,000 and the cost to produce each unit is \$250. How many units must be produced for the company to break even?

Let $x = \#$ units.

$C = 213{,}000 + 250x$

$R = 550x$

$C = R \Rightarrow 213{,}000 + 250x = 550x$

$\therefore 300x = 213{,}000$

$x = 710.$

$\therefore$ 710 units must be sold to break even

(b)[5] Sugar has supply equation $p = 1.4S - 0.6$ and demand equation $p = -2D + k$ where k is some value. Determine the value of k if the market price is $p = 2.9$.

At market price $p = 2.9$,

S is given by $2.9 = 1.4S - 0.6$

$\Rightarrow S = \frac{2.9 + 0.6}{1.4} = 2.5$

$\therefore D = 2.5$ also when $p = 2.9$.

So $2.9 = -2(2.5) + k$

$\Rightarrow k = 2.9 + 2(2.5) = \boxed{7.9}$