Question 1:

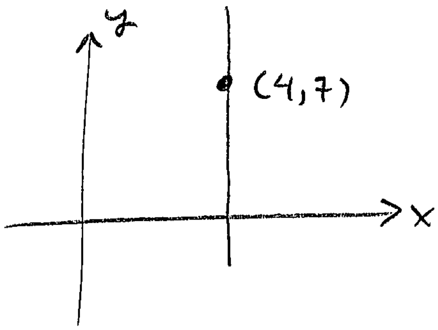
(a)[2] Determine the slope of the line through $(-5,-2)$ and $(-4,11)$.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{11-(-2)}{-4-(-5)} = \frac{13}{1} = \boxed{13}$$

(b)[2] Determine the slope of the line through $(-3,5)$ and $(2,5)$.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-5}{2-(-3)} = \frac{0}{5} = \boxed{0}$$

(c)[3] Determine an equation of the vertical line through $(4,7)$.



$$\boxed{x = 4.}$$

(d)[3] State the slope and y-intercept of the line $4x+3y=24$.

$$3y = -4x + 24$$

$$y = \frac{-4}{3}x + 8$$

$$\boxed{\therefore\ m = \frac{-4}{3},\ y\text{- intercept } (0,8).}$$

Question 2:

(a)[5] Determine an equation of the line through $(-5, 2)$ and parallel to the line through $(1, 2)$ and $(4, 3)$.

Line through $(1,2)$ & $(4,3)$

has slope $m = \frac{3-2}{4-1} = \frac{1}{3}$

$\therefore$ Line through $(-5,2)$ also has slope $m = \frac{1}{3}$ (since parallel),

so line has equation

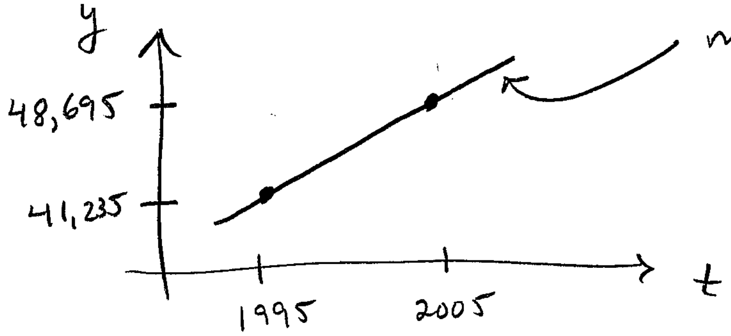
$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{1}{3}(x - (-5))$$

$$\boxed{y - 2 = \frac{1}{3}(x+5)}$$

or $\boxed{y = \frac{1}{3}x + \frac{11}{3}}$

(b)[5] In 1995 there were 41,235 shopping centres in the United States. By 2005 there were 48,695. Find a linear equation relating the year x to the number of shopping centres y, and use your equation to predict the year in which the number of shopping centres will reach 60,000.



$m = \frac{48,695 - 41,235}{2005 - 1995} = 746$

$\therefore$ Equation of line is $\boxed{y - 41235 = 746(t - 1995).}$

If $y = 60\,000$, $60000 - 41235 = 746(t - 1995)$

$$\Rightarrow t = \frac{60\,000 - 41235}{746} + 1995$$

$$\boxed{t \doteq 2020}$$

Question 3:

(a)[5] Eight hundred people attend a basketball game, and total ticket sales are \$3102 . If adult tickets are \$6 and student tickets are \$3, determine the number of each type of ticket sold.

Let x = # adults
y = # students.

$$\left.\begin{array}{ll} ① & x+y=800 \\ ② & 6x+3y=3102 \end{array}\right\} \quad \begin{array}{l} ① \Rightarrow y = 800 - x \\ ② \Rightarrow 6x + 3(800-x) = 3102 \end{array}$$

$$3x = 3102 - 2400$$

$$x = \frac{3102-2400}{3} = 234$$

$$\therefore\ y = 800 - 234 = 566.$$

$\boxed{\therefore \text{ 234 adult tickets and 566 student tickets were sold.}}$

(b)[5] A company manufactures a certain product and sells it for \$550 per unit. The fixed cost is \$213,000 and the cost to produce each unit is \$250. How many units must be produced for the company to break even?

Let x = # units.

$$C = 213{,}000 + 250x$$

$$R = 550x$$

$$C = R \Rightarrow 213{,}000 + 250x = 550x$$

$$300x = 213{,}000$$

$$x = 710$$

$\therefore$ 710 units must be sold to break even.

Question 4:

(a)[5] Sugar has supply equation $p = 1.4S - 0.6$ and demand equation $p = -2D + k$ where k is some value. Determine the value of k if the market price is $p = 2.9$.

At market price $p = 2.9$, S is given by $2.9 = 1.4S - 0.6$

$\Rightarrow S = \frac{2.9 + 0.6}{1.4} = 2.5$

$\therefore$ $D = 2.5$ also when $p = 2.9$.

So $2.9 = -2(2.5) + k$

$\Rightarrow k = 2.9 + 2(2.5) = \boxed{7.9}$

(b)[5] How many pounds of tea worth \$4.60 a pound should be mixed with tea worth \$6.50 a pound to get 10 pounds of blended tea worth \$5.74 a pound?

Let x = weight of tea worth 4.60 \$/lb,
y = weight of tea worth 6.50 \$/lb.

① $x + y = 10$

② $4.6x + 6.5y = (10)(5.74)$

① $\Rightarrow$ $y = 10 - x$

② $\Rightarrow$ $4.6x + 6.5(10 - x) = 57.4$

$4.6x + 65 - 6.5x = 57.4$

$1.9x = 7.6 \Rightarrow x = 4$

$\therefore y = 10 - x = 10 - 4 = 6.$

$\therefore$ 4 pounds of the first tea should be mixed with 6 pounds of the second.

Question 5 [10]: Solve the following system of equations **using either Gaussian or Gauss-Jordan elimination** (no credit will be given for using any other method). Use proper notation to state the row operations used at each step and clearly state the final solution.

$$x - 2y + 4z = 6$$
$$x + y + 13z = 6$$
$$-2x + 6y - z = -10$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 1 & 1 & 13 & 6 \\ -2 & 6 & -1 & -10 \end{array}\right]$$

$R_2 = (-1)r_1 + r_2$:
$R_3 = 2r_1 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 3 & 9 & 0 \\ 0 & 2 & 7 & 2 \end{array}\right]$$

$R_2 = (\frac{1}{3})r_2$:
$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 2 & 7 & 2 \end{array}\right]$$

$R_1 = (2)r_2 + r_1$:
$R_3 = (-2)r_2 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 10 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_2 = (-3)r_3 + r_2$:
$R_1 = (-10)r_3 + r_1$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\boxed{\therefore\ x = -14,\ y = -6,\ z = 2}$$