(1) [15] Find the maximum and minimum values of

$$z = 3x + 5y$$

subject to

$$\begin{aligned} 3x + 3y &\geq 9 \\ -x + 4y &\leq 12 \\ 4x - y &\leq 12 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Inequality	Line	test point	Test
$3x+3y \geq 9$	$3x+3y=9$	$(0,0)$	$0+0 \overset{?}{\geq} 9$: F
$-x+4y \leq 12$	$-x+4y=12$	$(0,0)$	$0+0 \overset{?}{\leq} 12$: T
$4x-y \leq 12$	$4x-y=12$	$(0,0)$	$0-0 \overset{?}{\leq} 12$: T

Corner points: • By inspection: $(0,3)$, $(3,0)$

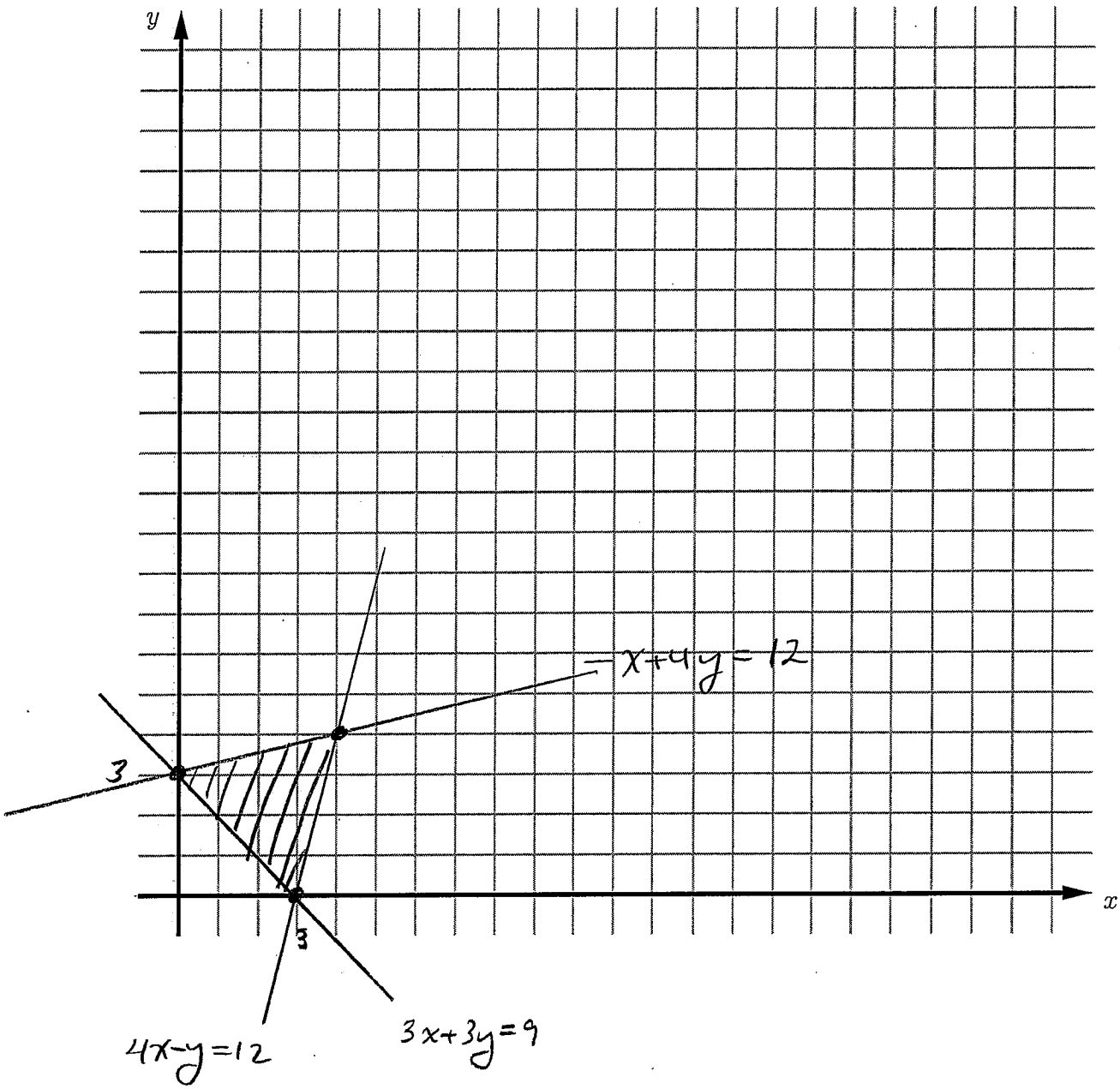
• Solving: ① $-x+4y=12$
② $4x-y=12$

② $\Rightarrow$ $y = 4x-12$
① $\Rightarrow$ $-x+4(4x-12)=12$
$15x = 60$
$x = 4$
$\therefore y = 4(4)-12 = 4$

$\Big\}$ $(4,4)$

C.P.	$z=3x+5y$
$(0,3)$	$z=15$
$(3,0)$	$z=9$
$(4,4)$	$z=32$

$\therefore$ z has a maximum of 32, minimum of 9.


y
x
x+4y=12
3
3
4x-y=12
3x+3y=9