**(1) [5]** Solve the following system of equation using Gaussian or Gauss-Jordan elimination.

$$6x + y = 8$$
$$x - 3y = -5$$
$$2x + y = 2$$

$$\left[\begin{array}{cc|c} 6 & 1 & 8 \\ 1 & -3 & -5 \\ 2 & 1 & 2 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $\left[\begin{array}{cc|c} \textcircled{1} & -3 & -5 \\ 6 & 1 & 8 \\ 2 & 1 & 2 \end{array}\right]$

$R_2 = (-6)r_1 + r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 19 & 38 \\ 0 & 7 & 12 \end{array}\right]$

$R_3 = (-2)r_1 + r_3$:

$R_2 = \frac{1}{19} r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & \textcircled{1} & 2 \\ 0 & 7 & 12 \end{array}\right]$

$R_3 = (-7)r_2 + r_3$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{array}\right]$ ← system has no solutions.

**(2) [5]** Compute $3\mathbf{BC} - 2\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & -2 & 5 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -2 & 1 \\ 3 & 5 \\ 5 & -2 \end{bmatrix}$$

$$3BC - 2A = 3\begin{bmatrix} 2 & -1 & 1 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & 5 \\ 5 & -2 \end{bmatrix} - 2\begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} -2 & -5 \\ 19 & -20 \end{bmatrix} - 2\begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -15 \\ 57 & -60 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ -6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -21 \\ 51 & -62 \end{bmatrix}$$

**(3) [5]** Determine $\mathbf{A}^{-1}$ where $\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}$.

$$\left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

$R_1 = (-1)r_1$:
$$\left[\begin{array}{cc|cc} 1 & -3 & -1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

$R_2 = (-3)r_1 + r_2$:
$$\left[\begin{array}{cc|cc} 1 & -3 & -1 & 0 \\ 0 & 10 & 3 & 1 \end{array}\right]$$

$R_2 = \frac{1}{10}r_2$:
$$\left[\begin{array}{cc|cc} 1 & -3 & -1 & 0 \\ 0 & 1 & \frac{3}{10} & \frac{1}{10} \end{array}\right]$$

$R_1 = (3)r_2 + r_1$:
$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{10} & \frac{3}{10} \\ 0 & 1 & \frac{3}{10} & \frac{1}{10} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}.$$