Question 1:

(a)[3] Suppose $f(x) = 2 - \int_2^{x^2+1} \frac{9}{1+t}\,dt$. Determine $f(1) - f'(1)$.

$$f(1) = 2 - \cancel{\int_2^2 \frac{9}{1+t}\,dt}^{\,0} = 2$$

$$f'(x) = -\left(\frac{9}{1+x^2+1}\right)(2x) \; ; \; f'(1) = -\left(\frac{9}{3}\right)(2) = -6.$$

$$\therefore \; f(1) - f'(1) = 2 - (-6) = \boxed{8}$$

(b)[3] Suppose you are using the definition of the definite integral (as a limit of Riemann sums) to evaluate $\int_0^\pi \sin(2x)\,dx$. Set up <u>*BUTDONOTEVALUATE*</u> the expression for the limit of the Riemann sums. Use subintervals of equal length to determine Δx and right endpoints of the subintervals to determine the x_i.

$$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$$

$$x_i = 0 + i\Delta x = \frac{i\pi}{n}.$$

$$\therefore \int_0^\pi \sin(2x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{2i\pi}{n}\right)\left(\frac{\pi}{n}\right).$$

(c)[4] If an animal's mass $w(t)$ is increasing at a rate of $w'(t) = t/(t^2+1)$ kilograms per year, what is the total mass gain over the next two years? (Assume that $t = 0$ corresponds to the present.)

$$w(2) - w(0) = \int_0^2 w'(t)\,dt$$

$$= \int_0^2 \frac{t}{t^2+1}\,dt$$

$$= \frac{1}{2}\left[\ln|t^2+1|\right]_0^2$$

$$= \boxed{\frac{\ln 5}{2} \text{ kg.}}$$

Question 2:

(a)[4] Evaluate $\int_0^1 t^3(1+t^4)^3\,dt$.

$u = 1+t^4$

$du = 4t^3$

$t=0 \Rightarrow u=1$

$t=1 \Rightarrow u=2$

$$\therefore \int_0^1 t^3(1+t^4)^3\,dt = \frac{1}{4}\int_1^2 u^3\,du = \frac{1}{4}\left[\frac{u^4}{4}\right]_1^2 = \frac{1}{16}(16-1) = \boxed{\frac{15}{16}}$$

(b)[3] Determine $\int \frac{e^{\tan x}}{\cos^2 x}\,dx = \int e^{\tan x}\sec^2 x\,dx$ $\left.\right\}$ $u = \tan x$, $du = \sec^2 x\,dx$

$$= \int e^u\,du = e^u + C = \boxed{e^{\tan x} + C}$$

(c)[3] Determine $\int \frac{\sqrt{1+\ln x}}{2x}\,dx$.

$u = 1+\ln x$

$du = \frac{1}{x}\,dx$

$$\therefore \int \frac{(1+\ln x)^{\frac{1}{2}}}{2x} = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\,\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = \boxed{\frac{1}{3}(1+\ln x)^{\frac{3}{2}} + C}$$

Question 3 [10 points]:

(a)[7] Evaluate $\int_0^\pi (x^2+1)\cos x\,dx = I$

$u = x^2+1$, $dv = \cos x\ dx$.
$du = 2x\,dx$, $v = \sin x$

$$\therefore I = \int_0^\pi u\,dv = uv\Big|_0^\pi - \int_0^\pi v\,du$$

$$= \cancel{(x^2+1)\sin x\Big|_0^\pi} \to 0 - 2\underbrace{\int_0^\pi x\sin x\,dx}$$

$u = x$, $dv = \sin x\,dx$
$du = dx$, $v = -\cos x\,dx$

$$= -2\left[-x\cos x\Big|_0^\pi + \int_0^\pi \cos x\,dx\right]$$

$$= -2\pi + \cancel{\sin x\Big|_0^\pi} \to 0$$

$$= \boxed{-2\pi}$$

(b)[3] Determine $\int \sin^2(\pi x)\,dx$.

$u = \pi x$
$du = \pi\,dx$

$$\int \sin^2(\pi x)\,dx = \frac{1}{\pi}\int \sin^2 u\,du$$

$$= \frac{1}{\pi}\int \frac{1}{2} - \frac{\cos(2u)}{2}\,du$$

$$= \frac{1}{2\pi}u - \frac{1}{4}\sin(2u) + C$$

$$= \frac{1}{2\pi}\pi x - \frac{1}{4}\sin(2\pi x) + C$$

$$= \frac{x}{2} - \frac{\sin(2\pi x)}{4} + C.$$

Question 4 [10 points]: Determine $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$.

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\therefore I = \int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

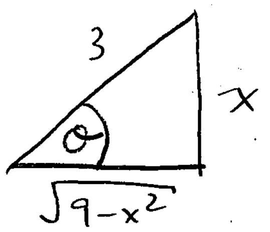
$$= \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta}\, 3\cos\theta\, d\theta$$

$$= \int \frac{\cancel{3}\cos\theta}{\cancel{9}\sin^2\theta}\, \cancel{3}\cos\theta\, d\theta$$

$$= \int \cot^2\theta\, d\theta$$

$$= \int \csc^2\theta - 1\, d\theta$$

$$= -\cot\theta - \theta + C.$$



$$\boxed{\therefore\ I = -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C}$$

Question 5 [10 points]: Determine $\int \frac{x+3}{x(x^2-1)}\,dx$.

$$I = \int \frac{x+3}{x(x-1)(x+1)}\,dx.$$

$$\frac{x+3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$= \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)}.$$

$$= \frac{(A+B+C)x^2 + (B-C)x + (-A)}{x(x-1)(x+1)}.$$

$$\therefore \left.\begin{aligned} A+B+C &= 0 \\ B-C &= 1 \\ -A &= 3 \end{aligned}\right\} \quad \therefore \begin{aligned} &A = -3 \\ &B = 1+C \\ &\therefore -3+(1+C)+C = 0 \\ &\qquad C = 1 \\ &\quad \therefore B = 2 \end{aligned}$$

$$\therefore I = \int \frac{-3}{x} + \frac{2}{x-1} + \frac{1}{x+1}\,dx$$

$$= \boxed{-3\ln|x| + 2\ln|x-1| + \ln|x+1| + C.}$$