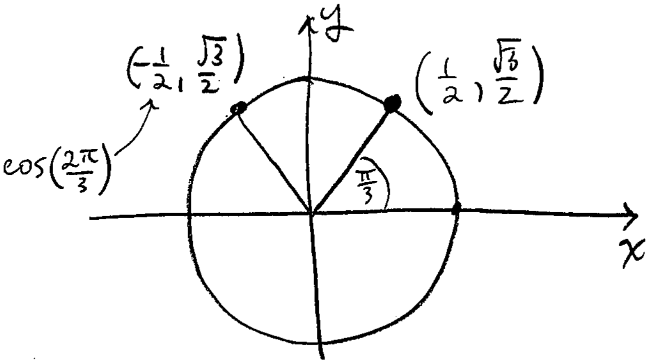
Question 1:

(a)[3] Evaluate arccos $(-1/2)$.



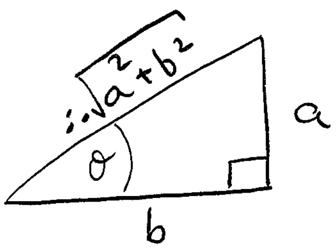
$\arccos\left(\frac{-1}{2}\right)$ = angle $\theta \in [0, \pi]$ such that $\cos\theta = \frac{-1}{2}$

$= \boxed{\frac{2\pi}{3}}.$

(b)[3] Simplify $\sin[\arctan(a/b)]$. Your final answer should not contain any trigonometric or inverse trigonometric functions

let $\theta = \arctan\left(\frac{a}{b}\right)$

$\therefore \tan\theta = \frac{a}{b}$:



$\therefore \sin\left[\arctan\left(\frac{a}{b}\right)\right]$

$= \sin\theta$

$= \boxed{\frac{a}{\sqrt{a^2+b^2}}}$

(c)[4] Determine an equation of the tangent line to the graph of $f(x) = \arcsin(x) + \arccos(x)$ at the point where $x = 0$.

$y = f(0) = \arcsin(0) + \arccos(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = 0$, so slope $m = f'(0) = 0$.

$\therefore$ Equation is $y - \frac{\pi}{2} = 0(x-0)$

i.e. $\boxed{y = \frac{\pi}{2}}$

Question 2:

(a)[5] Find all values of x for which $\operatorname{sech}(\ln x) = 1$.

$$\operatorname{sech}(\ln x) = 1$$

$$\Rightarrow \frac{1}{\cosh(\ln x)} = 1$$

$$\Rightarrow \cosh(\ln x) = 1$$

$$\Rightarrow \frac{e^{\ln x} + e^{-\ln x}}{2} = 1$$

$$\Rightarrow x + \frac{1}{x} = 2$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 = 0$$

$$\Rightarrow \boxed{x = 1}$$

(b)[5] Let $y = (\sinh(x) + \cosh(x))^2$. Compute and simplify $y''' - 8y$ (Hint: avoid expanding the square terms.)

$$y' = 2(\sinh x + \cosh x)(\cosh x + \sinh x) = 2(\sinh x + \cosh x)^2$$

↑
NOTICE!

$$\therefore y'' = 4(\sinh x + \cosh x)^2$$

$$y''' = 8(\sinh x + \cosh x)^2$$

$$\therefore y''' - 8y = \boxed{0}$$

Question 3:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x\to 0} \frac{x\cos(x)}{\ln(1+x)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{\cos(x) - x\sin(x)}{\frac{1}{1+x}}$$

$$= \frac{1-0}{1}$$

$$= \boxed{1}$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x\to\infty} \frac{e^{4x} - 1 - 4x}{x^2} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{4e^{4x} - 4}{2x} \sim \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{16e^{x}}{2}$$

$$= \boxed{\infty}$$

Question 4:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) \sim \text{"}\infty-\infty\text{"}$$

$$= \lim_{x\to 1^+} \frac{x\ln x - x + 1}{(x-1)\ln x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 1^+} \frac{\ln x + \cancel{x\frac{1}{x}} \cancel{-1}}{\ln x + (x-1)\frac{1}{x}} \sim \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}}$$

$$= \boxed{\frac{1}{2}}$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x\to 0^+}(\cosh(x))^{1/x} \sim \text{"}1^\infty\text{"}$$

$$(\cosh(x))^{\frac{1}{x}} = e^{\frac{1}{x}\ln[\cosh x]}.$$

$$\lim_{x\to 0^+} \frac{\ln[\cosh x]}{x} \sim \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0^+} \frac{\left(\frac{\sinh x}{\cosh x}\right)}{1}$$

$$= 0$$

$$\therefore \lim_{x\to 0^+}[\cosh(x)]^{\frac{1}{x}} = e^0 = \boxed{1}$$

Question 5:

(a)[5] An object initially $s(0) = 2$ m above the surface of the moon is projected vertically upward with an initial velocity of $v(0) = 3$ m/s. Using the fact that acceleration due to gravity on the moon is $a(t) = -1.6\ \text{m/s}^2$, derive the formula for $s(t)$, the height of the object above the moon's surface at time t seconds.

$a(t) = -1.6$

$\therefore v(t) = -1.6t + C_1$

$v(0) = 3 \Rightarrow C_1 = 3$

$\therefore v(t) = -1.6t + 3$

$\therefore s(t) = -\frac{1.6t^2}{2} + 3t + C_2$

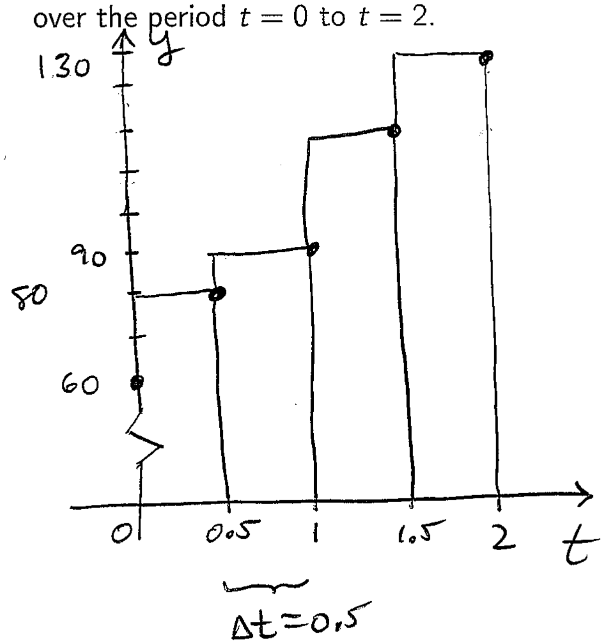
$s(0) = 2 \Rightarrow C_2 = 2$

$\boxed{\therefore s(t) = -0.8t^2 + 3t + 2}$

(b)[5] The rate of increase of a growing town's population is determined at five points in time, resulting in the following data:

t (years)	0	0.5	1	1.5	2
$r(t)$ (people per year)	60	80	90	110	130

Assuming that $r(t)$ is an increasing function of time, give an upper estimate of the population increase over the period $t = 0$ to $t = 2$.



Increase in population

$\approx R_4$

$= (80)(0.5) + (90)(0.5) + (110)(0.5) + (130)(0.5)$

$= \boxed{205 \text{ people}}$