 Name: SOLUTIONS.

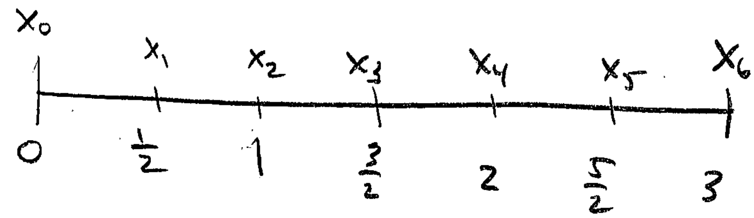
(1) [8] Use the trapezoid rule on $n = 6$ subintervals to approximate

$$\int_0^3 \frac{1}{1+x^2}\,dx$$

You may leave your answer in a "calculator ready" form; that is, you may leave it as an unsimplified sum of fractions.

$$\Delta x = \frac{3-0}{6} = \frac{1}{2}$$

$$f(x) = \frac{1}{1+x^2}$$



$$T_6 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2f(x_5) + f(x_6)\right]$$

$$= \frac{1}{4}\left[\frac{1}{1+0^2} + \frac{2}{1+\left(\frac{1}{2}\right)^2} + \frac{2}{1+1^2} + \frac{2}{1+\left(\frac{3}{2}\right)^2} + \frac{2}{1+2^2} + \frac{2}{1+\left(\frac{5}{2}\right)^2} + \frac{1}{1+3^2}\right]$$

$$= \boxed{\frac{1}{4}\left[1 + \frac{8}{5} + 1 + \frac{8}{13} + \frac{2}{5} + \frac{8}{29} + \frac{1}{10}\right]}$$

$$= \frac{18817}{15080}$$

$$\approx 1.25$$

 Name: SOLUTIONS.

(2) [7] Evaluate the improper integral

$$\int_1^\infty \frac{\ln x}{x^2}\,dx$$

making proper use of any required limits.

$$I = \int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2}\,dx.$$

For $\int \frac{\ln x}{x^2}\,dx$, let $u = \ln x$ $\quad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{-1}{x}$

$$\therefore \int \frac{\ln x}{x^2}\,dx = \frac{-\ln x}{x} - \int \frac{-1}{x}\,\frac{1}{x}\,dx$$

$$= \frac{-\ln x}{x} - \frac{1}{x} + C$$

$$\therefore I = \lim_{t\to\infty} \left[\frac{-\ln x}{x} - \frac{1}{x}\right]_1^t$$

$$= \lim_{t\to\infty}\left[\frac{-\ln t - 1}{t}\right] - \frac{\cancel{-\ln 1}^{\,0} - 1}{1}$$

$$\overset{H}{=} \lim_{t\to\infty} \frac{\left(-\frac{1}{t}\right)}{1} + 1$$

$$= \boxed{1}$$