

(1) [5] Evaluate $\int_1^2 x\sqrt{x-1} dx$.

$$u = x-1 \Rightarrow x = u+1$$

$$du = dx$$

$$x=1 \Rightarrow u=0$$

$$x=2 \Rightarrow u=1$$

$$\begin{aligned} \therefore \int_1^2 x\sqrt{x-1} dx &= \int_0^1 (u+1)u^{1/2} du \\ &= \int_0^1 u^{3/2} + u^{1/2} du \\ &= \frac{2}{5} [u^{5/2}]_0^1 + \frac{2}{3} [u^{3/2}]_0^1 = \frac{2}{5} + \frac{2}{3} = \boxed{\frac{16}{15}} \end{aligned}$$

(2) [5] Determine $\int re^{r/2} dr$.

$$u = r, \quad dv = e^{r/2} dr$$

$$du = dr, \quad v = 2e^{r/2}$$

$$\begin{aligned} \therefore I &= \int u dv = uv - \int v du \\ &= 2re^{r/2} - \int 2e^{r/2} dr \end{aligned}$$

$$= \boxed{2re^{r/2} - 4e^{r/2} + C}$$

(3) [5] Determine $\int \tan^3(x) \sec(x) dx$.

$$I = \int \tan^3(x) \sec(x) dx$$

$$= \int \tan^2(x) \sec(x) \tan(x) dx$$

$$= \int (\sec^2(x) - 1) \sec(x) \tan(x) dx$$

$$u = \sec(x)$$

$$du = \sec(x) \tan(x) dx$$

$$\therefore I = \int (u^2 - 1) du$$

$$= \frac{u^3}{3} - u + C$$

$$= \boxed{\frac{\sec^3(x)}{3} - \sec(x) + C}$$