Question 1: Evaluate the following limits, if they exist. If a limit does not exist but is ∞ or $-\infty$, state which with an explanation of your answer.

(a)[3] $\lim_{x\to -3^-} \dfrac{\cancel{x+3}}{\cancel{|x+3|}}$ $\quad \dfrac{|x-3|}{x+3} \begin{matrix}\to 6 \\ \to 0^-\end{matrix}$

$$\therefore \lim_{x\to -3^-} \frac{|x-3|}{x+3} = \boxed{-\infty}$$

(b)[4] $\lim_{x\to -\infty} \dfrac{9x^4 + x}{2x^4 + 5x^2 - 1}$ $\quad \begin{matrix}\div x^4 \\ \div x^4\end{matrix}$

$$= \lim_{x\to -\infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^4}}$$

$$= \boxed{\frac{9}{2}}$$

(c)[3] $\lim_{x\to\infty} \dfrac{\sqrt{9x^2 - x} - 3x}{1} \cdot \dfrac{\sqrt{9x^2-x} + 3x}{\sqrt{9x^2-x} + 3x}$

$$= \lim_{x\to\infty} \frac{\cancel{9x^2} - x \cancel{-9x^2}}{\sqrt{9x^2-x} + 3x}$$

$$= \lim_{x\to\infty} \frac{-\cancel{x}}{\cancel{x}\left(\sqrt{9 - \frac{1}{x}} + 3\right)}$$

$$= \boxed{\frac{-1}{6}}$$

Question 2:

(a)[3] Determine an equation of the tangent line to the curve $y = \sqrt{x+4} - \cos(x)$ at the point where $x = 0$.

At $x=0$, $y = \sqrt{0+4} - \cos(0) = 2-1 = 1$

$y' = \frac{1}{2}(x+4)^{-\frac{1}{2}} + \sin(x)$

$\therefore y'\big|_{x=0} = \frac{1}{2}(4)^{-\frac{1}{2}} + \sin(0) = \frac{1}{4}$

$\therefore y-1 = \frac{1}{4}(x-0) \Rightarrow \boxed{y = \frac{1}{4}x+1}$

(b)[3] After t seconds a projectile launched from ground level reaches a height of $s(t) = 20t - gt^2$ metres where g is a positive constant. At what time t does the projectile reach it's maximum height? (Your answer may contain the constant g.)

Solve $s'(t) = 0$

$20 - 2gt = 0$

$t = \frac{20}{2g} = \boxed{\frac{10}{g}\ s}$

(c)[4] There are two values of x at which the tangent line to $y = x^3$ is parallel to the tangent line to $y = \frac{3}{2}x^2 + 6x + 1$. Find these two values of x.

Solve $\frac{d}{dx}\left[x^3\right] = \frac{d}{dx}\left[\frac{3}{2}x^2 + 6x + 1\right]$

$3x^2 = 3x + 6$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2,\ x = -1}$

Question 3: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $y = \dfrac{\sqrt{t}}{3+2\sqrt{t}}$ $= \dfrac{t^{1/2}}{3+2t^{1/2}}$

$$y' = \frac{(3+2t^{1/2})(\frac{1}{2}t^{-\frac{1}{2}}) - t^{1/2}(t^{-1/2})}{(3+2t^{1/2})^2}$$

$$= \boxed{\frac{(3+2t^{1/2})(\frac{1}{2}t^{-\frac{1}{2}}) - 1}{(3+2t^{1/2})^2}}$$

(b)[3] $f(x) = x^7 + \sqrt{7}x - \dfrac{1}{\pi+1}$

$$\boxed{f'(x) = 7x^6 + \sqrt{7}}$$

(c)[4] $g(x) = \left(x^5 - \dfrac{x^2}{2}\right)\tan(x)$

$$\boxed{g'(x) = (5x^4 - x)\tan(x) + \left(x^5 - \frac{x^2}{2}\right)\sec^2(x)}$$

Question 4: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $f(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

$$\boxed{f'(x) = 3\left(\frac{3x^2-2}{2x+3}\right)^2\left[\frac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2}\right]}$$

(b)[3] $y = \cos\left(\theta + \dfrac{1}{\theta}\right)$

$$\boxed{y' = -\sin\left(\theta+\frac{1}{\theta}\right)\left(1-\frac{1}{\theta^2}\right)}$$

(c)[4] $g(x) = x\sqrt{\sec(\pi x)} = x\left[\sec(\pi x)\right]^{\frac{1}{2}}$

$$\boxed{g'(x) = \left[\sec(\pi x)\right]^{\frac{1}{2}} + x\,\frac{1}{2}\left[\sec(\pi x)\right]^{-\frac{1}{2}}\sec(\pi x)\tan(\pi x)\cdot\pi}$$

Question 5:

(a)[5] Determine an equation of the tangent line to the following curve at the point $(-1, 2)$:

$$x^2y^3 - 12 = x^3y^2$$

$$\frac{d}{dx}\left[x^2y^3 - 12\right] = \frac{d}{dx}\left[x^3y^2\right]$$

$$2xy^3 + x^2 \cdot 3y^2 y' = 3x^2y^2 + x^3\, 2yy'$$

at $(-1,2)$: $(2)(-1)(2)^3 + (-1)^2(3)(2)^2 y' = (3)(-1)^2(2)^2 + (-1)^3(2)(2)y'$

$$-16 + 12y' = 12 - 4y'$$

$$y' = \frac{28}{16} = \frac{7}{4}$$

$$\boxed{\therefore \quad y - 2 = \frac{7}{4}(x+1)}$$

(b)[5] Are there any values of $x > 0$ at which $y = \sin(x - \sin(x))$ has horizontal tangents? If so, find at least one such x. If not, explain why.

$$y' = 0 \Rightarrow \cos(x - \sin(x)) \cdot \underbrace{(1 - \cos(x))}_{} = 0$$

$$1 - \cos(x) = 0$$

$$\cos(x) = 1$$

$$x = 0, \pm 2\pi, \pm 4\pi, \ldots$$

$\boxed{\text{So yes, } y = \sin(x - \sin(x)) \text{ has horizontal tangents at } x = 0, \pm 2\pi, \pm 4\pi, \ldots}$

* <u>note</u>: y' is ~~also~~ zero where $x - \sin x$ is an odd multiple of $\frac{\pi}{2}$, but solving for such x is difficult.