Question 1: Evaluate the following limits, if they exist. If a limit does not exist but is ∞ or $-\infty$, state which with an explanation of your answer.

(a)[3] $\lim_{x\to-\infty} \frac{9x^4+x}{2x^4+5x^2-1}$ $\begin{array}{l} \div x^4 \\ \div x^4 \end{array}$

$$= \lim_{x\to-\infty} \frac{9+\frac{1}{x^3}}{2+\frac{5}{x^2}-\frac{1}{x^4}}$$

$$= \boxed{\frac{9}{2}}$$

(b)[4] $\lim_{x\to\infty} \sqrt{9x^2-x}-3x$

$$= \lim_{x\to\infty} \frac{\sqrt{9x^2-x}-3x}{1} \cdot \frac{\sqrt{9x^2-x}+3x}{\sqrt{9x^2-x}+3x}$$

$$= \lim_{x\to\infty} \frac{9x^2-x-9x^2}{\sqrt{9x^2-x}+3x}$$

$$= \lim_{x\to\infty} \frac{-\cancel{x}}{\cancel{x}\left(\sqrt{9-\frac{1}{x}}+3\right)} = \boxed{\frac{-1}{6}}$$

(c)[3] $\lim_{x\to-3^-} \cancel{\frac{x+3}{|x+3|}}$ $\frac{|x-3|}{x+3}$ $\begin{array}{l} \to 6 \\ \to 0^- \end{array}$

$$\therefore \lim_{x\to-3^-} \frac{|x-3|}{x+3} = \boxed{-\infty}$$

Question 2:

(a)[3] After t seconds a projectile launched from ground level reaches a height of $s(t) = 20t - gt^2$ metres where g is a positive constant. At what time t does the projectile reach it's maximum height? (Your answer may contain the constant g.)

Solve $\cancel{s}'(t) = 0$

$$20 - 2gt = 0$$

$$t = \frac{20}{2g} = \boxed{\frac{10}{g}\ s.}$$

(b)[3] Determine an equation of the tangent line to the curve $y = \sqrt{x+4} - \cos(x)$ at the point where $x = 0$.

At $x=0$, $y = \sqrt{0+4} - \cos(0) = 2 - 1 = 1$

$$y' = \frac{1}{2}(x+4)^{-\frac{1}{2}} + \sin(x).$$

$$\therefore y'\Big|_{x=0} = \frac{1}{2}(4)^{-\frac{1}{2}} + \sin(0) = \frac{1}{4}$$

$$\therefore \quad y - 1 = \frac{1}{4}(x-0) \Rightarrow \boxed{y = \frac{1}{4}x + 1}$$

(c)[4] There are two values of x at which the tangent line to $y = x^3$ is parallel to the tangent line to $y = \frac{3}{2}x^2 + 6x + 1$. Find these two values of x.

Solve $\frac{d}{dx}\left[x^3\right] = \frac{d}{dx}\left[\frac{3}{2}x^2 + 6x + 1\right]$

$$3x^2 = 3x + 6$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\boxed{x = 2, \quad x = -1}$$

Question 3: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $f(x) = x^7 + \sqrt{7}x - \dfrac{1}{\pi + 1}$

$$\boxed{f'(x) = 7x^6 + \sqrt{7}.}$$

(b)[3] $y = \dfrac{\sqrt{t}}{2 + 3\sqrt{t}}$ $= \dfrac{t^{1/2}}{2 + 3t^{1/2}}$

$$y' = \frac{(2+3t^{1/2})(\frac{1}{2}t^{-\frac{1}{2}}) - t^{1/2}(\frac{3}{2}t^{-\frac{1}{2}})}{(2+3t^{1/2})^2}$$

(c)[4] $g(x) = \left(x^5 - \dfrac{x^2}{2}\right)\tan(x)$

$$\boxed{g'(x) = (5x^4 - x)\tan(x) + \left(x^5 - \frac{x^2}{2}\right)\sec^2(x)}$$

Question 4: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $y = \sin\left(\theta - \frac{1}{\theta}\right)$

$$y' = \cos\left(\theta - \frac{1}{\theta}\right)\left(1 + \frac{1}{\theta^2}\right)$$

(b)[3] $f(x) = \left(\frac{3x^2-2}{2x+3}\right)^3$

$$f'(x) = 3\left(\frac{3x^2-2}{2x+3}\right)^2\left[\frac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2}\right]$$

(c)[4] $g(x) = x\sqrt{\sec(\pi x)}$ $= x[\sec(\pi x)]^{1/2}$

$$g'(x) = [\sec(\pi x)]^{\frac{1}{2}} + x\,\tfrac{1}{2}[\sec(\pi x)]^{-\frac{1}{2}}\cdot\sec(\pi x)\tan(\pi x)\cdot\pi$$

Question 5:

(a)[5] Determine an equation of the tangent line to the following curve at the point $(-1, 2)$:

$$x^2y^3 - x^3y^2 = 12$$

$$\frac{d}{dx}\left[x^2y^3 - x^3y^2\right] = \frac{d}{dx}[12]$$

$$2xy^3 + x^2 3y^2 y' - 3x^2y^2 - x^3 2yy' = 0.$$

at $(-1,2)$: $(2)(-1)(2)^3 + (-1)^2(3)(2)^2y' - (3)(-1)^2(2)^2 - (-1)^3(2)(2)y' = 0$

$$-16 + 12y' - 12 + 4y' = 0$$

$$y' = \frac{28}{16} = \frac{7}{4}$$

$$\boxed{\therefore \quad y - 2 = \frac{7}{4}(x+1)}$$

(b)[5] Are there any values of $x > 0$ at which $y = \sin(x - \sin(x))$ has horizontal tangents? If so, find at least one such x. If not, explain why.

$$y' = 0 \Rightarrow \cos(x - \sin(x)) \cdot \underbrace{(1 - \cos(x))} = 0.$$

$$1 - \cos(x) = 0$$

$$\Rightarrow \cos(x) = 1$$

$$\Rightarrow x = 0, \pm 2\pi, \pm 4\pi, \ldots$$

$$\boxed{\text{So yes: } y = \sin(x - \sin(x)) \text{ has horizontal tangents at } x = 0, \pm 2\pi, \pm 4\pi, \ldots}$$

* note: y' is also zero at x for which $x - \sin x$ is an odd multiple of $\frac{\pi}{2}$, but these x values are difficult to determine.