Question 1: Evaluate the following limits, if they exist. If a limit does not exist but is ∞ or $-\infty$, state which with an explanation of your answer.

(a)[3] $\displaystyle \lim_{x \to -\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - 1}$

(b)[4] $\displaystyle \lim_{x \to \infty} \sqrt{9x^2 - x} - 3x$

(c)[3] $\displaystyle \lim_{x \to -3^-} \frac{x + 3}{|x + 3|}$

Question 2:

(a)[3] After t seconds a projectile launched from ground level reaches a height of $s(t) = 20t - gt^2$ metres where g is a positive constant. At what time t does the projectile reach it's maximum height? (Your answer may contain the constant g.)

(b)[3] Determine an equation of the tangent line to the curve $y = \sqrt{x+4} - \cos(x)$ at the point where $x = 0$.

(c)[4] There are two values of x at which the tangent line to $y = x^3$ is parallel to the tangent line to $y = \dfrac{3}{2}x^2 + 6x + 1$. Find these two values of x.

Question 3: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $f(x) = x^7 + \sqrt{7}\,x - \dfrac{1}{\pi + 1}$

(b)[3] $y = \dfrac{\sqrt{t}}{2 + 3\sqrt{t}}$

(c)[4] $g(x) = \left(x^5 - \dfrac{x^2}{2} \right) \tan(x)$

Question 4: Determine the derivative of each of the following functions (it is not necessary to simplify final answers):

(a)[3] $y = \sin\left(\theta - \dfrac{1}{\theta}\right)$

(b)[3] $f(x) = \left(\dfrac{3x^2 - 2}{2x + 3}\right)^3$

(c)[4] $g(x) = x\sqrt{\sec(\pi x)}$

Question 5:

(a)[5]   Determine an equation of the tangent line to the following curve at the point $(-1, 2)$:

$$x^2y^3 - x^3y^2 = 12$$

(b)[5]   Are there any values of $x > 0$ at which $y = \sin(x - \sin(x))$ has horizontal tangents? If so, find at least one such x. If not, explain why.